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Ranking of Fuzzy Numbers by using Scaling Method

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ABSTRACT

In this paper, we presented for the first time a multidimensional scaling approach to find the scaling as well as the ranking of triangular fuzzy numbers. Each fuzzy number was represented by a row in a matrix, and then found the configuration points (scale points) which represent the fuzzy numbers in R^2. Since these points are not uniquely determined, then we presented different techniques to reconfigure the points to compare them with other methods. The results showed the ability of ranking fuzzy numbers.

Keywords: Fuzzy numbers, Ranking, Scaling method, Multidimensional scaling

1. Introduction

Ranking fuzzy numbers is a necessary step in making decision. Most of the presented method for ranking is not able to discriminate fuzzy numbers. Hence it is not possible to order them because they are partially ordered and cannot compare directly like real numbers [14].

In order to rank fuzzy numbers, It is possible to convert them into real numbers and compared them by a ranking function. After the work of Zadeh (1965) [9], many ranking methods have been introduced. First work was proposed by Jain [15], since then many procedures are presented to do this aim. Some of ranking methods were reviewed in [8]. Chen introduced maximizing and minimizing set for ranking fuzzy numbers [18]. The idea of the mean was proposed by Dubois and Prade to rank fuzzy numbers [5]. Statistically, Lee and Li used probability measure to compare fuzzy numbers [6].

Allahviranloo, et al., applied a new distance measure and the generalized Hausdorff distance between generalized fuzzy numbers [10]. PhaniBushan and RaoPeddia used maximizing and minimizing set on the triangular fuzzy numbers which depend on the centroid point [13]. Ebrahimnejad et al. used a signed distance ranking to order interval-valued fuzzy numbers [1].

In this paper, we used the classical multidimensional scaling to rank fuzzy numbers which is presented for the first time. We treated each fuzzy number as a row matrix to construct our matrix and used MDS techniques to find the configuration points (scale points). These points represent the fuzzy number themselves in R^2. Since these points are not uniquely presented, this is the nature of MDS method. We presented different techniques to reconfigure the scale points to compare the results with other methods.

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2. Basic Concepts and Definitions

Zadeh [9] dealt with problems that have source of vagueness. A fuzzy set is an extension of a crisp set by defining partial membership [23].

Definition (1) [16]

A fuzzy set $\tilde{A}$ of universe set X is the set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, where $\mu_{\tilde{A}}(x): X \to [0,1]$, where $\mu_{\tilde{A}}(x)$ is the membership function for the fuzzy set.

Definition (2) [8]

A fuzzy number $\tilde{A}$ is a fuzzy subset in which it is normal and convex with the following membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}^{L}(x) & if \;\; a \le x \le b \\ w & if \;\; b \le x \le c \\ \mu_{\tilde{A}}^{R}(x) & if \;\; c \le x \le d \\ 0 & otherwise, \end{cases}$$

where $0 < w \le 1$ is a constant, a, b, c, d are real numbers, and $\mu_{\tilde{A}}^{L}: [a, b] \to [0, w]$, $\mu_{\tilde{A}}^{L}\ [c, d] \to [0, w]$ are two strictly monotonic and continuous functions from R to be closed interval [0,w]. It is customary to write fuzzy number as $\tilde{A} = (a, b, c, d, w)$, if $w = 1$, then $\tilde{A} = (a, b, c, d, 1)$ is normalized fuzzy number, otherwise $\tilde{A}$ is non-normal fuzzy number.

Definition (3) [18]

A triangular fuzzy number (TFN) $\tilde{A}$ is a fuzzy number with a membership function $\mu_{\tilde{A}}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & \text{otherwise,} \end{cases}$$

which can be denoted as a triplet (a_1, a_2, a_3).

Definition (4) [7]

$\tilde{A}$ is said to be a triangular fuzzy number if its membership function is:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & if\ \ a_1 < x < a_2 \\ \frac{x-a_3}{a_3-a_2}, & if\ \ a_2 < x < a_3 \\ 0, & otherwise \end{cases}$$

with $a_1 \le a_2 \le a_3$, and can be represented by $\tilde{A} = (a_1, a_2, a_3)$, then calculate the mean of $\tilde{A}$ as:

$$E(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}.$$

3. Ranking Fuzzy Numbers

First, the synthesis of Au-crown/carboxylic was planned through the pathways shown in Scheme 1. Au is the soft acid and cysteine is soft bases (donor), therefore a strong covalent bond (Au-SR) occurs between Au and cysteine. Eventually, the N-terminal cysteine residues to react with ether crown aldehyde group to give imines [22]. Due to the fact that the carboxylic acid functional group is present in the cysteine ligand, so by placing crown ether on the surface of gold nanoparticles, it is modified with two groups.

4. Mathematics of Metric Multidimensional Scaling [16]

This method starts with an $(n \times n)$ matrix $D = (\delta_{ij})$ called distance matrix. The aim is to find n points in k dimensions such that the interpoint distances d_{ij} in the k dimensions are approximately equal to the values of δ_{ij} in D. Ideally $k = 2$. The steps are as follows

- $A = (a_{ij}) = (-\frac{1}{2}\delta_{ij}^2)$, is an $(n \times n)$ matrix, where δ_{ij} is the ijth element of D.
- $B = (b_{ij})$, is an $(n \times n)$ matrix, where
- $b_{ij} = a_{ij} - \bar{a}_{i.} - \bar{a}_{.j} + \bar{a}$ ... and
- $\bar{a}_{i.} = \sum_{j=1}^{n} a_{ij}/n$, $\bar{a}_{.j} = \sum_{i=1}^{n} a_{ij}/n$. So

$$B = \left(I - \tfrac{1}{n}J\right) A \left(I - \tfrac{1}{n}J\right), \tag{1}$$

and

$$B = \text{V}\Lambda\text{V}', \tag{2}$$

where V is the matrix eigenvectors of B and Λ is the diagonal matrix of eigenvalues of B. Also it can be written as

$$B = \text{V}_1\Lambda_1\text{V}_1'$$

$$= \text{V}_1\Lambda_1^{1/2}\Lambda_1^{1/2}\text{V}_1'$$

$= ZZ'$,

where

$$Z = \text{V}_1\Lambda_1^{1/2}\left(\sqrt{\lambda_1}\text{v}_1, \sqrt{\lambda_2}\text{v}_2, \dots, \sqrt{\lambda_q}\text{v}_\text{q}\right)\begin{pmatrix} z_1' \\ z_2' \\ \cdot \\ \cdot \\ \cdot \\ z_n' \end{pmatrix} \tag{3}$$

is our scale points.

5. Different Techniques for Ranking

5. 1 Scaling Method S_1

Parandin and Fariborzi [12] introduced a new idea by finding a new point differs from zero, and use a distance function. Apply it to the our points we define $M = (\bar{z}_m, \bar{z}_p)$ such that $\bar{z}_m$ is maximum of z_{i1} and $\bar{z}_p$ is maximum of z_{i2}, for $i = 1,2,\dots,n$. Let $Z_{\tilde{A}_1} = (z_{11}, z_{12})$, $Z_{\tilde{A}_2} = (z_{21}, z_{22})$, …, $Z_{\tilde{A}_n} = (z_{n1}, z_{n2})$ be the scales of the fuzzy numbers.

Calculate the distance between M and $Z_{\tilde{A}_i}$ as follows,

$$R(Z_{\tilde{A}_i}, M) = \sqrt{(\bar{z}_M - z_{i1})^2 + \left(\bar{z}_p - z_{i2}\right)^2}\ .$$

If $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy numbers, then

1. $R\left(Z_{\tilde{A}_1}, M\right) < R\left(Z_{\tilde{A}_2}, M\right)$ implies that $\tilde{A}_1 > \tilde{A}_2$
2. $R(Z_{\tilde{A}_1}, M) > R(Z_{\tilde{A}_2}, M)$ implies that $\tilde{A}_1 < \tilde{A}_2$

5. 2 Scaling Method S_2

Allahviranloo and Saneifard [19] introduced the maximum crisp value τ_{max} as

$$\tau_{max} = max\ \{x | x\ \in Domain\ (\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)\},$$

and found a new point $(\tau_{max}, 0)$, then used a distance between centroid pinot and the new point.

Applying this idea, we construct the point $T = (\tau_{max}, 0)$, and the scaling points are $Z_{\tilde{A}_1} = (z_{11}, z_{12})$, $Z_{\tilde{A}_2} = (z_{21}, z_{22})$, …, $Z_{\tilde{A}_n} = (z_{n1}, z_{n2})$. The ranking function is the distance between T and $Z_{\tilde{A}_i}$ as follows:

$$R(Z_{\tilde{A}_i}, T) = \sqrt{(z_{i1} - \tau_{max})^2 + (z_{i2} - 0)^2}$$

If $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy numbers, then

1. $R(Z_{\tilde{A}_1}, T) < R(Z_{\tilde{A}_2}, T)$ implies that $\tilde{A}_1 > \tilde{A}_2$

2. $R(Z_{\tilde{A}_1},T) > R(Z_{\tilde{A}_2},T)$ implies that $\tilde{A}_1 < \tilde{A}_2$

5. 3 A Novel Proposed Method S_3

Here, we introduce a novel method by defining the maximum crisp value μ_{max} as follows:

$$\mu_{max} = max\{E(\tilde{A}_1), E(\tilde{A}_2), \dots, E(\tilde{A}_n)\},$$

where $E(\tilde{A}_i)$ is the mean value of fuzzy numbers (Definition 4). Construct the point $E = (\mu_{max},0)$ and $Z_{\tilde{A}_1} = (z_{11},\ z_{12})$, $Z_{\tilde{A}_2} = (z_{21},\ z_{22})$, …, $Z_{\tilde{A}_n} = (z_{n1},\ z_{n2})$, where $Z_{\tilde{A}_i}$ are the scaling points for $i = 1,2,..,n$.

Now for ranking, we calculate the distance between E and $\tilde{A}_i$ as follows:

$$R(\tilde{A}_i, E) = \sqrt{(z_{i1} - \mu_{max})^2 + (z_{i2} - 0)^2}$$

If $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy numbers, then

$R(Z_{\tilde{A}_1},E) < R(Z_{\tilde{A}_2},E)$ implies that $\tilde{A}_1 > \tilde{A}_2$

$R(Z_{\tilde{A}_1},E) > R(Z_{\tilde{A}_2},E)$ implies that $\tilde{A}_1 < \tilde{A}_2$

Proposition (1): If at least one of $z_{i1} > \mu_{max}$, then $\exists\ \beta > 0$, such that $z_{i1} - \beta \le \mu_{max}$ for each $i = 1,2,\dots,n$.

Proof:

Since $z_{i1} > \mu_{max}$, say i_0 then $z_{i_01} - \mu_{max} > 0$. Let $\beta = |z_{i_01} - \mu_{max}|$. So, $z_{i_01} - \beta \le \mu_{max}$. ■

For ranking fuzzy numbers, the new scale points is

$$Z^{new} = \begin{pmatrix} z_{11} - \beta & z_{12} \\ z_{21} - \beta & z_{22} \\ \vdots & \vdots \\ z_{n1} - \beta & z_{n2} \end{pmatrix}$$

5. 4 Shifting Scale Points Techniques

It is important to mention that if the scale points are not all in R^+, so we face some shortcomes when find the distance with $(0,0)$ point. One of our tasks is to fix it by the following preposition.

Let

$$Z = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \\ \vdots & \vdots \\ z_{i1} & z_{i2} \end{pmatrix}$$

Preposition (2): If at least one of $z_{i1} < 0$ or $z_{i2} < 0$, then $\exists\ \epsilon_x > 0$ and $\epsilon_y > 0$ respectively such that $z_{i1} + \epsilon_x$ and $z_{i2} + \epsilon_y$ are in $R^+, \forall i = 1,2,\dots,n$.

Proof:

The goal is to find such ϵ_x and ϵ_y. Let z_{i_01}=min(z_{i1}), $\forall i = 1,2,\dots,n$. For ϵ_x, z_{i_01}+$\epsilon_x > 0$, implies that $\epsilon_x > -z_{i_01}$, which is the lower bound of ϵ_x.

Also, let z_{i_01}+$\epsilon_x \le \mu_{max}$, implies that $\epsilon_x \le \mu_{max} - z_{i_01}$, which is the upper bound of ϵ_x, and μ_{max} is maximum mean of the fuzzy number $\tilde{A}_i$. Therefore $\epsilon_x \in (z_{i_01}, \mu_{max} - z_{i_01}]$.

For ϵ_y, let z_{i_02} =min$\{z_{i2}\}$, so z_{i_02}+$\epsilon_y > 0$, implies that $\epsilon_y > -z_{i_02}$, which is the lower bound

of ϵ_y. Again, let z_{i_02}+$\epsilon_y \le$ max$\{z_{i2}\}$, implies that $\epsilon_y \le$ max$\{z_{i2}\}, -z_{i_02}$, which is the upper bound of ϵ_y. Therefore $\epsilon_y \in (-z_{i_02}, \max\{z_{i2}\} - z_{i_02}]$. ■

So, in S_4, the new scale points is

$$Z^* = \begin{pmatrix} z_{11} + \epsilon_x & z_{12} + \epsilon_y \\ z_{21} + \epsilon_x & z_{22} + \epsilon_y \\ \vdots & \vdots \\ z_{n1} + \epsilon_x & z_{n2} + \epsilon_y \end{pmatrix}$$

Remark (1):

If all $z_{i1} > 0$, then no need to shift as well as for z_{i2}.

Remark (2):

For ranking, we choose the midpoint of ϵ_x and ϵ_y.

5. 5 A Novel Proposed Method S_4

Define the ranking function as $R\left(Z^*_{\tilde{A}_i}\right) = \sqrt{(z^*_{i1})^2 + (z^*_{i2})^2}$, for $\forall i$, and $R\left(Z^*_{\tilde{A}_i}\right)$ is the distance between point $Z^*_{\tilde{A}_i}$ and the original point.

If $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy numbers, then

$R(Z^*_{\tilde{A}_1}) < R(Z^*_{\tilde{A}_2})$ implies that $\tilde{A}_1 < \tilde{A}_2$

$R(Z^*_{\tilde{A}_1}) > R(Z^*_{\tilde{A}_2})$ implies that $\tilde{A}_1 > \tilde{A}_2$

Remark (3)

For all the proposed methods S_1, S_2, S_3, S_4. If the ranking function is equal, then it is possible to replace the position of the eigenvalues, to fix it.

5. 6 Computational Results

In this section, we will apply our proposed methods on several instances, where some of the aforementioned methods are unable to rank them. Let

Set 1: $\tilde{A} = (0.2, 0.3, 0.5)$, $\tilde{B} = (0.17, 0.32, 0.58)$ and $\tilde{C} = (0.25, 0.4, 0.7)$

Set 2: $\tilde{A} = (2, 4, 6)$, $\tilde{B} = (1, 5, 6)$ and $\tilde{C} = (3, 5, 6)$

Set 3: $\tilde{A} = (0.1, 0.2, 0.3)$, $\tilde{B} = (0.15, 0.26, 0.32)$ and $\tilde{C} = (0.15, 0.3, 0.4)$

Set 4: $\tilde{A} = (1, 2, 5)$, $\tilde{B} = (0, 3, 4)$ and $\tilde{C} = (2, 2.5, 3)$

Set 5: $\tilde{A} = (0, 1, 3)$, $\tilde{B} = (4, 5, 6)$ and $\tilde{C} = (7, 8, 9)$

Set 6: $\tilde{A} = (1, 1, 5)$, $\tilde{B} = (0.25, 1, 2)$ and $\tilde{C} = (1, 2, 9)$.

Table (1) shows the results of the proposed methods

Table 1: Results for the (set 1, set 2, set 3)

Authors	Fuzzy Number	Set 1	Set 2	Set 3
Cheng distance [4]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.5900 0.6040 0.6620	4.0311 4.0348 4.6943	0.5385 0.5636 0.5810
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Parandin and Fariborzi [12]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.3420 0.1170 0.0015	0.6672 0.6666 0.0206	0.0838 0.0400 0.0012
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Allahviranloo and Saneifard [19]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.6097 0.5955 0.5462	2.0615 2.0688 1.4271	0.5385 0.5320 0.5205
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Chu and Tsao [20]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.1620 0.1740 0.2190	2.0000 2.1176 2.3742	0.1000 0.1237 0.1437
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Asady and Zendehnam [3]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.2500 0.2275 0.3250	2.2500 1.2500 3.2500	0.1250 0.1650 0.1750
Results		$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Phani and Ravi [14]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.5180 0.5266 0.5847	4.1036 4.5924 4.3402	0.4591 0.4847 0.5080
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Yager [17]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.3250 0.3475 0.4375	4.0000 4.2500 4.7500	0.2000 0.2475 0.2875
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Wang and Luo [22]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.2925 0.3349 0.5047	0.6000 0.6500 0.7500	0.3333 0.4917 0.6294
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Chen [18]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.3706 0.3997 0.4828	0.5516 0.6140 0.6525	0.4098 0.5072 0.5753

Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Chang [21]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.0166 0.0267 0.3375	4.0000 8.0000 4.3333	0.0100 0.0133 0.0212
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$
S_1 method	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.2350 0.1597 0.0414	1.0000 2.2368 0.3333	0.1500 0.0902 0.0020
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
S_2 method	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.7996 0.7308 0.5705	6.0369 7.0079 5.0110	0.4734 0.4045 0.3234
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
S_3 method	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.5247 0.4563 0.2956	4.8790 5.8428 3.8477	0.3650 0.2964 0.2151
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
S_4 method	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.2130 0.2901 0.4422	4.3687 2.4394 4.4291	0.1534 0.2118 0.2993
Results		$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$

Table 2: Results for the (set 4, set 5, set 6)

Authors	Fuzzy Number	Set 4	Set 5	Set 6
Cheng distance [4]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	2.7071 2.3935 2.5495	1.4126 5.0249 8.0156	2.3655 1.1890 4.0310
Results		$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Parandin and Fariborzi [12]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	0.0666 0.3333 0.1700	6.6667 3.0000 0.0000	1.6704 2.9265 0.0000
Results		$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Allahviranloo and Saneifard [19]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	2.3500 2.6800 2.5200	7.6808 4.0311 1.1180	6.6779 7.9318 5.0249
Results		$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Chu and Tsao [20]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	1.2444 1.2444 1.2500	0.6222 2.5000 4.0000	0.9074 0.5310 2.0000
Results		$\tilde{A} \sim \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Asady and Zendehnam [3]	$\tilde{A}$ $\tilde{B}$ $\tilde{C}$	1.7500 0.2500 2.0625	0.5000 4.2500 7.2500	0.5000 2.0000 2.7500
Results		$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$

Phani and Ravi [14]	$\tilde{A}$	2.4051	1.1931	1.0576
	$\tilde{B}$	2.7296	5.0006	1.7178
	$\tilde{C}$	2.5279	8.0004	3.5599
Results		$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Yager [17]	$\tilde{A}$	2.5000	1.2500	2.0000
	$\tilde{B}$	2.5000	5.0000	1.0625
	$\tilde{C}$	2.5000	8.0000	3.500
Results		$\tilde{A} \sim \tilde{B} \sim \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Wang and Luo [22]	$\tilde{A}$	0.5000	0.1389	0.2000
	$\tilde{B}$	0.5000	0.5558	0.0929
	$\tilde{C}$	0.4667	0.8889	0.3714
Results		$\tilde{C} < \tilde{A} \sim \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Chen [18]	$\tilde{A}$	0.4720	0.2654	0.2933
	$\tilde{B}$	0.5280	0.5355	0.2178
	$\tilde{C}$	0.5000	0.7634	0.3881
Results		$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Chang [21]	$\tilde{A}$	1.3333	0.6666	0.0000
	$\tilde{B}$	3.5000	2.5000	0.4062
	$\tilde{C}$	0.3125	4.0000	2.0000
Results		$\tilde{C} < A < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
S_1 method	$\tilde{A}$	2.1213	11.5758	4.0833
	$\tilde{B}$	2.7385	5.1961	7.1384
	$\tilde{C}$	0.8660	0.0000	0.0578
Results		$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
S_2 method	$\tilde{A}$	4.1940	14.9900	9.3605
	$\tilde{B}$	5.9084	8.5997	12.3813
	$\tilde{C}$	5.1961	3.4165	5.2714
Results		$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
S_3 method	$\tilde{A}$	1.9344	13.9901	4.1029
	$\tilde{B}$	3.6026	7.6002	7.1125
	$\tilde{C}$	3.0183	2.4176	0.1707
Results		$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
S_4 method	$\tilde{A}$	2.5426	4.0226	4.5286
	$\tilde{B}$	1.4018	10.3936	1.6754
	$\tilde{C}$	3.6925	15.5823	8.6378
Results		$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$

In Table 1 and set 1, the proposed methods give $\tilde{A} < \tilde{B} < \tilde{C}$, as in most of the methods. For set 2, they give $\tilde{B} < \tilde{A} < \tilde{C}$, where, most of other methods give $\tilde{A} < \tilde{B} < \tilde{C}$, and for set 3, they give $\tilde{A} < \tilde{B} < \tilde{C}$ as in all the methods.

In Table 2, for set 4, the proposed methods are capable to recognize these numbers, S_1 , S_4 give $\tilde{B} < \tilde{A} < \tilde{C}$, and , S_2 , S_3 give $\tilde{B} < \tilde{C} < \tilde{A}$, whereas some of other methods couldn't rank the numbers such as Yager [17], and Chu and Tsao [20] . The other methods give different results.

For set 5, all the proposed methods and others give $\tilde{A} < \tilde{B} < \tilde{C}$, and For set 6, the proposed methods give and others give $\tilde{B} < \tilde{A} < \tilde{C}$ as most of the other methods.

4. Conclusion

In this paper we proposed several approaches to find a scale of triangular fuzzy numbers, and ranking them by scaling method. The idea is presented for the first time, and has a simple structure. The scale points can be reconfigured by many techniques to fit the original distances. An important property is that, this method ranks all the triangular fuzzy numbers, unless they are equal.

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