Ranking of Fuzzy Numbers by using Scaling Method

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1. Introduction

Ranking fuzzy numbers is a necessary step in making decision. Most of the presented method for ranking is not able to discriminate fuzzy numbers. Hence it is not possible to order them because they are partially ordered and cannot compare directly like real numbers. In order to rank fuzzy numbers, it is possible to convert them into real numbers and compared them by a ranking function. After the work of Zadeh (1965) [9], many ranking methods have been introduced. First work was proposed by Jain [15] since then many procedures are presented to do this aim. Some of ranking methods were reviewed in [8]. Chen introduced maximizing and minimizing set for ranking fuzzy numbers [18]. The idea of the mean was proposed by Dubois and Prade to rank fuzzy numbers [5]. Statistically, Lee and Li used probability measure to compare fuzzy numbers [6]. Allahviranloo, et al., applied a new distance measure and the generalized Hausdorff distance between generalized fuzzy numbers [10]. PhaniBushan and RaoPeddia used maximizing and minimizing set on the triangular fuzzy numbers which depend on the centroid point [13]. Ebrahimnejad et al. used a signed distance ranking to order interval-valued fuzzy numbers [11].

In this paper, we used the classical multidimensional scaling to rank fuzzy numbers which is presented for the first time. We treated each fuzzy number as a row matrix to construct our matrix and used MDS techniques to find the configuration points (scale points). These points represent the fuzzy number themselves in $R^2$. Since these points are not uniquely presented, this is the nature of MDS method. We presented different techniques to reconfigure the scale points to compare the results with other methods.

2. Basic Concepts and Definitions

Zadeh [8] dealt with problems that have source of vagueness. A fuzzy set is an extension of a crisp set by defining partial membership [23].

Definition (1) [16]

A fuzzy set $\tilde{A}$ of universe set $X$ is the set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))| x \in X\}$, where $\mu_{\tilde{A}}(x) : X \rightarrow [0,1]$, where $\mu_{\tilde{A}}(x)$ is the membership function for the fuzzy set.

Definition (2) [8]

A fuzzy number $\tilde{A}$ is a fuzzy subset in which it is normal and convex with the following membership function

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
\mu_1^A(x) & \text{if } a \leq x \leq b \\
w & \text{if } b \leq x \leq c \\
\mu_2^A(x) & \text{if } c \leq x \leq d \\
0 & \text{otherwise},
\end{cases}
$$

where $0 < w \leq 1$ is a constant, $a, b, c, d$ are real numbers, and $\mu_1^A : [a,b] \rightarrow [0,w]$, $\mu_2^A : [c,d] \rightarrow [0,w]$ are two strictly monotonic and continuous functions from $R$ to be closed interval $[0,w]$. It is customary to write fuzzy number as $\tilde{A} = (a, b, c, d, w)$, if $w = 1$, then $\tilde{A} = (a, b, c, d, 1)$ is normalized fuzzy number, otherwise $\tilde{A}$ is non-normal fuzzy number.
Definition (3) \[18\]
A triangular fuzzy number (TFN) $\tilde{A}$ is a fuzzy number with a membership function $\mu_{\tilde{A}}(x)$
\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
a_2, & x = a_2 \\
\frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}, \end{cases}
\]
which can be denoted as a triplet $(a_1, a_2, a_3)$.

Definition (4) \[7\]
$\tilde{A}$ is said to be a triangular fuzzy number if its membership function is:
\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 < x < a_2 \\
\frac{x-a_2}{a_3-a_2}, & a_2 < x < a_3 \\
0, & \text{otherwise}, \end{cases}
\]
with $a_1 \leq a_2 \leq a_3$, and can be represented by $\tilde{A} = (a_1, a_2, a_3)$, then calculate the mean of $\tilde{A}$ as:
\[
E(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}.
\]

3. Ranking Fuzzy Numbers

First, the synthesis of Au-crown/carboxylic was planned through the pathways shown in Scheme 1. Au is the soft acid and cysteine is soft bases (donor), therefore a strong covalent bond (Au-SR) occurs between Au and cysteine. Eventually, the N-terminal cysteine residues to react with ether crown aldehyde group to give imines \[22\]. Due to the fact that the carboxylic acid functional group is present in the cysteine ligand, so by placing crown ether on the surface of gold nanoparticles, it is modified with two groups.

4. Mathematics of Metric Multidimensional Scaling \[16\]
This method starts with an $(n \times n)$ matrix $D = (\delta_{ij})$ called distance matrix. The aim is to find $n$ points in $k$ dimensions such that the interpoint distances $d_{ij}$ in the $k$ dimensions are approximately equal to the values of $\delta_{ij}$ in $D$. Ideally $k = 2$. The steps are as follows

- $A = (a_{ij}) = (-\frac{1}{2} \delta_{ij})$, is an $(n \times n)$ matrix, where $\delta_{ij}$ is the $ij$th element of $D$.
- $B = (b_{ij})$, is an $(n \times n)$ matrix, where
  - $b_{ij} = a_{ij} - \bar{a}_i - \bar{a}_j + \bar{a} \ldots$ and
  - $\bar{a}_i = \sum_{j=1}^{n} a_{ij}/n, \bar{a}_j = \sum_{i=1}^{n} a_{ij}/n$. So
\[
B = \left(I - \frac{1}{n}\right) A \left(I - \frac{1}{n}\right),
\]
and
\[
B = V \Lambda V',
\]
where $V$ is the matrix eigenvectors of $B$ and $\Lambda$ is the diagonal matrix of eigenvalues of $B$. Also it can be written as

\[
B = V_1 \Lambda_1 V_1' = V_1 \Lambda_1^{1/2} \Lambda_1^{1/2} V_1' = \mathbb{Z} Z',
\]

where
\[
Z = V_1 \Lambda_1^{1/2} \left(\sqrt{\lambda_1 v_1}, \sqrt{\lambda_2 v_2}, \ldots, \sqrt{\lambda_q v_q}\right)
\]
is our scale points.

5. Different Techniques for Ranking

5.1 Scaling Method $S_1$

Parandin and Fariborzi \[12\] introduced a new idea by finding a new point differs from zero, and use a distance function. Apply it to the our points we define $M = (z_m, z_p)$ such that $z_m$ is maximum of $z_{11}$ and $z_p$ is maximum of $z_{12} \ldots$ for $i = 1, 2, \ldots, n$. Let $\tilde{A}_i = (z_{11}, z_{12}), \tilde{A}_2 = (z_{21}, z_{22}) \ldots, \tilde{A}_n = (z_{n1}, z_{n2})$ be the scales of the fuzzy numbers.

Calculate the distance between $M$ and $\tilde{A}_i$ as follows,
\[
R(\tilde{A}_i, M) = \sqrt{(z_m - z_{11})^2 + (z_p - z_{12})^2}.
\]
If $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy numbers, then
1. $R(\tilde{A}_1, M) < R(\tilde{A}_2, M)$ implies that $\tilde{A}_1 > \tilde{A}_2$
2. $R(\tilde{A}_1, M) > R(\tilde{A}_2, M)$ implies that $\tilde{A}_1 < \tilde{A}_2$

5.2 Scaling Method $S_2$

Allahviranloo and Saneifard \[19\] introduced the maximum crisp value $\tau_{\text{max}}$ as
\[
\tau_{\text{max}} = \max \{x | x \in \text{Domain} (\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n)\},
\]
and found a new point $(\tau_{\text{max}} , 0)$, then used a distance between centroid pinot and the new point.

Applying this idea, we construct the point $T = (\tau_{\text{max}}, 0)$, and the scaling points are $\tilde{A}_1 = (z_{11}, z_{12}), \tilde{A}_2 = (z_{21}, z_{22}) \ldots, \tilde{A}_n = (z_{n1}, z_{n2})$. The ranking function is the distance between $T$ and $\tilde{A}_i$ as follows:
\[
R(\tilde{A}_i, T) = \sqrt{(z_{1i} - \tau_{\text{max}})^2 + (z_{2i} - 0)^2}
\]
If $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy numbers, then
1. $R(\tilde{A}_1, T) < R(\tilde{A}_2, T)$ implies that $\tilde{A}_1 > \tilde{A}_2$
2. If  \( R(Z_{A_1}, T) > R(Z_{A_2}, T) \) implies that  \( A_1 < A_2 \)

5.3 A Novel Proposed Method \( S_3 \)

Here, we introduce a novel method by defining the maximum crisp value  \( \mu_{\text{max}} \) as follows:

\[
\mu_{\text{max}} = \max\{ E(\tilde{A}_1), E(\tilde{A}_2), ..., E(\tilde{A}_n) \},
\]

where  \( E(\tilde{A}_i) \) is the mean value of fuzzy numbers (Definition 4). Construct the point  \( E = (\mu_{\text{max}} \cdot 0) \) and  \( Z_{A_1} = (z_{11}, z_{12}), Z_{A_2} = (z_{21}, z_{22}), ..., Z_{A_n} = (z_{n1}, z_{n2}) \), where  \( Z_{A_i} \) are the scaling points for  \( i = 1, 2, ..., n \).

Now for ranking, we calculate the distance between  \( E \) and  \( \tilde{A}_i \) as follows:

\[
R(\tilde{A}_i, E) = \sqrt{(z_{11} - \mu_{\text{max}})^2 + (z_{12} - 0)^2}
\]

If  \( \tilde{A}_1 \) and  \( \tilde{A}_2 \) are two fuzzy numbers, then

\[
R(Z_{A_1}, E) < R(Z_{A_2}, E) \quad \text{implies that} \quad \tilde{A}_1 > \tilde{A}_2
\]

\[
R(Z_{A_1}, E) > R(Z_{A_2}, E) \quad \text{implies that} \quad \tilde{A}_1 < \tilde{A}_2
\]

**Proposition (1):** If at least one of  \( z_{i1} > \mu_{\text{max}} \), then  \( \exists \beta > 0 \), such that  \( z_{i1} - \beta \leq \mu_{\text{max}} \) for each  \( i = 1, 2, ..., n \).

**Proof:**

Since  \( z_{i1} > \mu_{\text{max}} \), say  \( i_0 \) then  \( z_{i_0} - \mu_{\text{max}} > 0 \). Let  \( \beta = |z_{i_0} - \mu_{\text{max}}| \). So,  \( z_{i_0} - \beta \leq \mu_{\text{max}} \).

For ranking fuzzy numbers, the new scale points is

\[
Z^* = \begin{pmatrix}
  z_{11} + \epsilon_x & z_{12} + \epsilon_y \\
  z_{21} + \epsilon_x & z_{22} + \epsilon_y \\
  \vdots & \vdots \\
  z_{n1} + \epsilon_x & z_{n2} + \epsilon_y
\end{pmatrix}
\]

**Remark (1):**

If all  \( z_{i1} > 0 \), then no need to shift as well as for  \( z_{i2} \).

**Remark (2):**

For ranking, we choose the midpoint of  \( \epsilon_x \) and  \( \epsilon_y \).

5.5 A Novel Proposed Method \( S_4 \)

Define the ranking function as  \( R(Z_{A_1}) = \sqrt{(z_{11}^*)^2 + (z_{12}^*)^2} \), for  \( \forall i \), and  \( R(Z_{A_1}) = \) the distance between point  \( Z_{A_1}^* \) and the original point.

If  \( \tilde{A}_1 \) and  \( \tilde{A}_2 \) are two fuzzy numbers, then

\[
R(Z_{A_1}) < R(Z_{A_2}) \quad \text{implies that} \quad \tilde{A}_1 < \tilde{A}_2
\]

\[
R(Z_{A_1}) > R(Z_{A_2}) \quad \text{implies that} \quad \tilde{A}_1 > \tilde{A}_2
\]

**Remark (3):**

For all the proposed methods  \( S_1, S_2, S_3, S_4 \). If the ranking function is equal, then it is possible to replace the position of the eigenvalues, to fix it.
5.6 Computational Results

In this section, we will apply our proposed methods on several instances, where some of the aforementioned methods are unable to rank them. Let

Set 1: \( A = (0.2, 0.3, 0.5), \ B = (0.17, 0.32, 0.58) \) and \( C = (0.25, 0.4, 0.7) \)

Set 2: \( A = (2, 4, 6), \ B = (1, 5, 6) \) and \( C = (3, 5, 6) \)

Set 3: \( A = (0.1, 0.2, 0.3), \ B = (0.15, 0.26, 0.32) \) and \( C = (0.15, 0.3, 0.4) \)

Set 4: \( A = (1, 2, 5), \ B = (0.3, 4) \) and \( C = (2, 2.5, 3) \)

Set 5: \( A = (0, 1, 3), \ B = (4, 5, 6) \) and \( C = (7, 8, 9) \)

Set 6: \( A = (1, 1, 5), \ B = (0.25, 1, 2) \) and \( C = (1, 2, 9) \).

Table (1) shows the results of the proposed methods

<table>
<thead>
<tr>
<th>Authors</th>
<th>Fuzzy Number</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cheng distance [4]</td>
<td>( \tilde{A} )</td>
<td>0.5900</td>
<td>4.0311</td>
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<td></td>
<td>( \tilde{B} )</td>
<td>0.6040</td>
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<td></td>
<td>( \tilde{C} )</td>
<td>0.6620</td>
<td>4.6943</td>
<td>0.5810</td>
</tr>
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<td>Parandin and Fariborzi [12]</td>
<td>( \tilde{A} )</td>
<td>0.3420</td>
<td>0.6672</td>
<td>0.0838</td>
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<td>( \tilde{B} )</td>
<td>0.1170</td>
<td>0.6666</td>
<td>0.0400</td>
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<tr>
<td></td>
<td>( \tilde{C} )</td>
<td>0.0015</td>
<td>0.0206</td>
<td>0.0012</td>
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<td>Allahviranloo and Saneifard [19]</td>
<td>( \tilde{A} )</td>
<td>0.6097</td>
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</tr>
<tr>
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<td>( \tilde{C} )</td>
<td>0.5462</td>
<td>1.4271</td>
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<td>Chu and Tsao [20]</td>
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<td>( \tilde{C} )</td>
<td>0.2190</td>
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<td>0.1437</td>
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<td>Asady and Zendehnam [3]</td>
<td>( \tilde{A} )</td>
<td>0.2500</td>
<td>2.2500</td>
<td>0.1250</td>
</tr>
<tr>
<td></td>
<td>( \tilde{B} )</td>
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</tr>
<tr>
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<td>( \tilde{C} )</td>
<td>0.3250</td>
<td>3.2500</td>
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</tr>
<tr>
<td>Phani and Ravi [14]</td>
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<td>0.5180</td>
<td>4.1036</td>
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<tr>
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<td>( \tilde{B} )</td>
<td>0.5266</td>
<td>4.5924</td>
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<td>( \tilde{C} )</td>
<td>0.5847</td>
<td>4.3402</td>
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<td>Yager [17]</td>
<td>( \tilde{A} )</td>
<td>0.3250</td>
<td>4.0000</td>
<td>0.2000</td>
</tr>
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<td>0.3475</td>
<td>4.2500</td>
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<tr>
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<td>( \tilde{C} )</td>
<td>0.4375</td>
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<td>Wang and Loo [22]</td>
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<td></td>
<td>( \tilde{B} )</td>
<td>0.3349</td>
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<tr>
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<td>( \tilde{C} )</td>
<td>0.5047</td>
<td>0.7500</td>
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</tr>
<tr>
<td>Chen [18]</td>
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<td>0.3706</td>
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<td>( \tilde{B} )</td>
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<td>( \tilde{C} )</td>
<td>0.4828</td>
<td>0.6525</td>
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<td>Results</td>
<td>$\tilde{A} &lt; B &lt; \tilde{C}$</td>
<td>$\tilde{A} &lt; \tilde{B} &lt; \tilde{C}$</td>
<td>$\tilde{A} &lt; B &lt; \tilde{C}$</td>
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<tr>
<td>---------</td>
<td>----------------</td>
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<td>----------------</td>
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</tr>
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<td>Chang $^{[21]}$</td>
<td>$\tilde{A}$</td>
<td>0.0166</td>
<td>4.0000</td>
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<td>8.0000</td>
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<td>$\tilde{C}$</td>
<td>0.3375</td>
<td>4.3333</td>
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<td>S$_1$ method</td>
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<td>$\tilde{B}$</td>
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<td>2.3668</td>
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<td>$\tilde{C}$</td>
<td>0.0414</td>
<td>0.3333</td>
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<td>S$_2$ method</td>
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<td>S$_3$ method</td>
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Table 2: Results for the (set 4, set 5, set 6)

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<td>2.7071</td>
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<td>Results</td>
<td>$\tilde{B} &lt; \tilde{C} &lt; \tilde{A}$</td>
<td>$\tilde{A} &lt; \tilde{B} &lt; \tilde{C}$</td>
<td>$\tilde{B} &lt; \tilde{A} &lt; \tilde{C}$</td>
<td></td>
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<tr>
<td>Parandin and Fariborzi $^{[12]}$</td>
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<td>0.0666</td>
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<td>Results</td>
<td>$B &lt; \tilde{C} &lt; \tilde{A}$</td>
<td>$\tilde{A} &lt; B &lt; \tilde{C}$</td>
<td>$\tilde{B} &lt; \tilde{A} &lt; \tilde{C}$</td>
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<td>Allahviranloo and Saneifard $^{[19]}$</td>
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<td>1.1180</td>
<td>5.0249</td>
</tr>
<tr>
<td>Results</td>
<td>$\tilde{B} &lt; \tilde{C} &lt; \tilde{A}$</td>
<td>$\tilde{A} &lt; B &lt; \tilde{C}$</td>
<td>$\tilde{B} &lt; \tilde{A} &lt; \tilde{C}$</td>
<td></td>
</tr>
<tr>
<td>Chu and Tsao $^{[20]}$</td>
<td>$\tilde{A}$</td>
<td>1.2444</td>
<td>0.6222</td>
<td>0.9074</td>
</tr>
<tr>
<td></td>
<td>$\tilde{B}$</td>
<td>1.2444</td>
<td>2.5000</td>
<td>0.5310</td>
</tr>
<tr>
<td></td>
<td>$\tilde{C}$</td>
<td>1.2500</td>
<td>4.0000</td>
<td>2.0000</td>
</tr>
<tr>
<td>Results</td>
<td>$\tilde{A} &lt; \tilde{B} &lt; \tilde{C}$</td>
<td>$\tilde{A} &lt; B &lt; \tilde{C}$</td>
<td>$\tilde{B} &lt; \tilde{A} &lt; \tilde{C}$</td>
<td></td>
</tr>
<tr>
<td>Asady and Zendehnam $^{[3]}$</td>
<td>$\tilde{A}$</td>
<td>1.7500</td>
<td>0.5000</td>
<td>0.5000</td>
</tr>
<tr>
<td></td>
<td>$\tilde{B}$</td>
<td>0.2500</td>
<td>4.2500</td>
<td>2.0000</td>
</tr>
<tr>
<td></td>
<td>$\tilde{C}$</td>
<td>2.0625</td>
<td>7.2500</td>
<td>2.7500</td>
</tr>
<tr>
<td>Results</td>
<td>$\tilde{B} &lt; \tilde{A} &lt; \tilde{C}$</td>
<td>$\tilde{A} &lt; \tilde{B} &lt; \tilde{C}$</td>
<td>$\tilde{A} &lt; \tilde{B} &lt; \tilde{C}$</td>
<td></td>
</tr>
</tbody>
</table>
For set 5, all the proposed methods and others give $\tilde{A} < \tilde{B} < \tilde{C}$, and for set 6, the proposed methods give and others give $\tilde{B} < \tilde{A} < \tilde{C}$ as most of the other methods.

4. Conclusion

In this paper we proposed several approaches to find a scale of triangular fuzzy numbers, and ranking them by scaling method. The idea is presented for the first time, and has a simple structure. The scale points can be reconfigured by many techniques to fit the original distances. An important property is that, this method ranks all the triangular fuzzy numbers, unless they are equal.
References


