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Original Article

Multivariate CUSUM Daubechies Discrete Wavelet Transformation Coefficients Charts for Quality Control

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ABSTRACT

The multivariate CUSUM chart was used to control and monitor several measurable qualitative characteristics of production processes through the accumulated sum of Mahalanobis distance. In this article, new charts correspond to the CUSUM chart based on wavelet analysis. The discrete wavelet transform of the Daubechies wavelet of order 40 was used to obtain approximation coefficients (proportional to the sum of observations) at each level and detail coefficients (proportional to the differences of observations) at each level. The approximation coefficients were used to calculate the cumulative sum statistic of Mahalanobis distance to construct the MCUSUM Daubechies wavelet approximate coefficients chart and to find the optimal reference value for target average run length, and the detail coefficients were used to calculate the cumulative sum statistic of Mahalanobis distance values and to construct the MCUSUM Daubechies wavelet detail coefficients chart to find the optimal reference value for target average run length. Therefore, two charts were obtained, the first of which is used to control the averages of the qualitative properties of the produced material, and the second is used to control the standard deviation of the qualitative properties. The proposed charts were more efficient than conventional charts while maintaining average run length, depending on the simulation study and real data for the Erbil Steel company.

Keywords: Quality Control Processes, Multivariate CUSUM Charts, Average Run Length, Daubechies Wavelet, Approximate and detailed coefficients.

1 Introduction

Quality management focuses on maintaining the value of products delivered to clients and encompasses quality assurance, control, and improvement. It is crucial across various manufacturing, healthcare, and government industries. Quality control, a key component, involves monitoring industrial processes to ensure output meets consumer standards [1]. With the rise of automated inspection and process control, especially in engineering and medical fields, the importance of quality control has surged [2]. Many researchers now focus on control charts and techniques that predict industrial issues and offer solutions. Quality-related problems have evolved with advancements in data collection, leading to more complex multivariate challenges [3]. Research continues to expand on classic quality control methods, with univariate control charts traditionally assessing single quality characteristics. However, relying on one characteristic can be misleading in real-world scenarios, where multiple quality attributes exist. Effective monitoring must encompass all production outputs with distinct characteristics, posing difficulties with classical control charts due to their extensive data handling [4]. As production scales, interpreting control charts becomes increasingly complex, necessitating approaches that account for data correlations. While various effective techniques exist, their complexity can hinder practical application, motivating researchers to investigate simpler methods suitable for lower-level staff [5].

In Industry 4.0, advanced quality control techniques have emerged, including statistical methods using univariate and multivariate statistics. Traditional quality control methods like control charts are evolving to keep pace with technology [6]. CUSUM is a suitable modern approach for detecting outliers and signaling out-of-control situations. This technique serves as an early monitoring system that helps organizations quickly identify issues, reducing errors and ensuring product quality [7]. Traditional methods cannot integrate real-time data effectively, which makes modern techniques necessary. Wavelet Transformation promotes data privileges, providing multicollinearity through eight orthogonal Daubechies wavelets, obtaining approximation and detail coefficients. Incorporating wavelet transforms into the control chart assists in the first stage of out-of-control situations [8]. The study presents the Multivariate CUSUM chart using wavelet transformation

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coefficients as the post determinations, structuring outcomes related to data coefficient shifts and edited slope factors for h and k values. The paper is crucial for multivariate discrete wavelet coefficients in monitoring [9].

In recent years, growing attention has been directed toward integrating wavelet transformations into industrial process monitoring systems. Majeed et al. (2023) introduced a wavelet-based statistical control chart designed to detect faults in spur gear systems, demonstrating their strong capability in identifying subtle operational shifts [10]. Similarly, another 2024 study proposed a kernel-based multivariate CUSUM control chart, which achieved remarkable efficiency in reducing Average Run Length (ARL) values when detecting abrupt changes in process performance [11]. These contributions highlight the practical advantages of combining advanced statistical control techniques with wavelet analysis, supporting the current study's approach of developing high-order Daubechies-based MCUSUM charts to enhance sensitivity to minor and gradual process shifts.

This study distinguishes itself from prior research by utilizing a high-order Daubechies wavelet transform to improve the accuracy of detecting process shifts in MCUSUM control charts, thereby achieving greater sensitivity to small and gradual changes in process behavior. Therefore, this article suggests new multivariate CUSUM (MCUSUM) control charts relying on wavelet analysis to check and control the amount characteristics of the steps of outcomes. The discrete wavelet transforms, especially the Daubechies wavelet of order 40, are carried out to calculate approximation and detail coefficients from the data. These coefficients are finally utilized to compute the cumulative sum statistic of Mahalanobis distance, finalizing two purpose MCUSUM charts: one for controlling the means of qualitative properties and the second one for controlling the standard deviations. The new charts highlight the greatness of efficiency over conventional ways while maintaining the desired average run length.

2 Methods and Materials

2. 1 Quality Control Processes

Quality control guarantees products match standards and client assumptions in manufacturing and services [12]. It includes notifications and measurements to find quality deviations, with statistical techniques like Multivariate CUSUM, Daubechies Discrete Wavelet Transformation Coefficients Charts assisting in recognizing subtle process shifts for temporary mixing up [13, 14]. Pattern checking is crucial to quality control, mostly needing modifications in manufacturing processes [15]. Control charts, exclusively Shewhart and process control charts, are tremendously utilized across sectors, throughout automotive, electrical, and medical industries, to determine data correlations and weigh production variations. The need for multivariable datasets has increased beyond univariate applications [16, 17]. Monitoring these datasets presents significant challenges due to interdependent variables and unevenly distributed groups or classifications. Contemporary ways depend on assumptions for process signal finding, which may not capture data irregularity well [18, 19]. New research tries to develop statistical techniques to actively check local events in irregular data, familiarizing variations with minimal overall effect [20].

2. 2 Multivariate CUSUM Charts

It simply describes the innovation and importance of Multivariate CUSUM (MCUSUM) charts as an extension of conventional control chart strategies, evolving from univariate applications to advanced multivariate forms [21, 22]. CUSUM, short for cumulative sum, is very important in quality control for determining subtle shifts in process parameters over time. Completely different from outdated Shewhart charts, which may miss minor deviations, the CUSUM chart can find signals as early as the 50th observation [23]. The MCUSUM statistic, which promote monitoring across correlated results, is calculated as:

$$S_t = \max(0, S_{t-1} + \sqrt{(Y_t - \mu)^t \Sigma^{-1} (Y_t - \mu)} - k) \quad (1)$$

$S_0 = 0$, Y_t is the observed vector, μ is the target mean, Σ is the covariance matrix, and k is the sensitivity threshold. Applied in auto-factories to check out-of-control behavior on various panel sides, MCUSUM charts have been innovated to applications like cumulative sum control charts employing Daubechies wavelet transformation for multi-attribute monitoring [24]. So, it will further highlight univariate and multivariate CUSUM charts, highlight theoretical determinations in change-point issues [25], and simplify the design recognizing of MCUSUM for identifying process shifts in multiple correlated attributes [26].

2. 3 Average Run Length

The Average Run Length (ARL) is a key performance criterion utilized to create control charts. ARL measures the predicted number of samples or observations taken before a control chart signals an out-of-control condition. For the provided control chart, the ARL can be represented as:

$$ARL = \frac{1}{\alpha} \quad (2)$$

where $\alpha \neq 0$ and represents the possibility of a wrong alarm. For an in-control process, typically, ARL_0 refers to the average number of points before a false alarm when the process is in control, and it is commonly set to 200. In contrast, ARL_1 measures the average number of points until detecting an actual shift; here, lower values are preferred to ensure quick detection of process changes [27].

The ARL criterion is also used in multivariate control charts, like the Exponentially Weighted Moving Average (EWMA) charts, to measure the response to transformations in joint and marginal parameters. We can simply tell ARL is used as a quality metric pointing to the impact of the control chart to determine shifts in process parameters, providing insights into the performance of control charts across varied process conditions [28, 29].

2. 4 Daubechies Wavelet

The Daubechies wavelet is one of the greatest impactful discrete wavelet transforms (DWTs), widely used across engineering disciplines, particularly in signal processing applications such as image coding and resolution [30]. This wavelet is good enough at the detail level, let any level can be used, though higher levels yield more refined outcomes. For accurate signal reconstruction, it is fundamental to stay at a steady level [31, 32].

In the Daubechies wavelet, signal decomposition is gained through filters, also well known as background filters, initialized by scalars γ_0 and γ_1 [33]. Convolution and decimation generate wavelet coefficients, merging at specified scales and time points to produce the transform's output. The formula representing a Daubechies wavelet transform ultimately relies on recursive filters defined by the filter coefficients γ_i, satisfying orthogonality conditions [34]. For example, a Daubechies wavelet $\psi(t)$ can be described by a recursive relation that starts with scaling functions $\phi(t)$ such that:

$$\phi(t) = \sum_{k=0}^{N-1} \gamma_k \ \phi(2t - k) \tag{3}$$

and the associated wavelet function $\psi(t)$ is defined by:

$$\psi(t) = \sum_{k=0}^{N-1} (-1)^k \gamma_{N-1-k} \ \phi(2t - k) \tag{4}$$

where N denotes the number of coefficients, specific to the Daubechies wavelet family. Wavelet coefficients derived through these processes discover variations in data as the wavelet modifies across scales and time locations, a privilege valuable in quality control applications [35, 36].

2. 5 Discrete Wavelet Transformation

The Discrete Wavelet Transform (DWT) is often utilized in signal processing because of its compact support and vanishing moments, making it influential for representing data across multiple scales [37]. Its power to minimize mistakes from floating-point operations enhances the steadiness of algorithms, making it completely relevant for analyzing non-stationary signals [38]. DWT is specifically important in missions like denoising, compression, and image processing, as it focuses on signal details at various scales [39]. It is also beneficial in quality control and industrial monitoring, including thorough data transformation and compression [40, 41]. The orthogonality and thresholding features of DWT simplify data reduction while preserving steady signal properties [42]. These attributes make DWT precious in various industries for tasks like data decomposition and noise suppression [43, 44].

2. 6 Proposed Charts

It is usable to fabricate new charts depending on the discrete wavelet transform of the Daubechies wavelet of order 40, clearly to make a chart of approximation and detail coefficients for the Daubechies wavelet for the MCUSUM through the following steps:

Step 1. Assume that X is a data matrix, where the columns represent several variables (p), and the rows represent the observations (n) for even numbers n. Perform a discrete wavelet transform for the Daubechies wavelet of order (N = 40) for each variable of the data matrix to produce (n/2 +N -1) of the approximate and detailed coefficients (two partitions) as in the following formulas:

$$A(s) \equiv V_{j,i} = \sum_{l=0}^{L-1} g_l V_{j-1,2i+1-l \bmod Nj-1} \tag{5}$$

$$D(s) \equiv W_{j,i} = \sum_{l=0}^{L-1} h_l V_{j-1,2i+1-l \bmod Nj-1} \tag{6}$$

Define $V_0 = x$ and x is the observation vector of length (n) and set $j = 1$ (the level), input to the j^{th} stage of the pyramid algorithm is V_{j-1} (which is full-band), and related to frequencies $[0, 1/2^j]$ in x. Half-band filters for $i = 0, 1, \dots, N_j-1$. Formula 1 represents the approximation coefficients (first part), scale or factor function A(s) and s = 1, 2, …, p with (n/2+N-1) coefficients, which are proportional to the qualitative characteristics of observations summation at each level *L*. In contrast, formula 6 represents (second part) the detail coefficients D(s), the mother or wavelet function with (n/2+N-1) coefficients, which are proportional to the differences (variance) of the observations of the qualitative characteristic.

Step 2. Repeat the calculation of the discrete wavelet transform for the Daubechies wavelet of order 40 for each column of the data matrix X. The following matrices of multivariate approximation and detail coefficients are obtained:

$$XA = [A(1)\ A(2) \dots A(p)] \tag{7}$$

$$XD = [D(1)\ D(2) \dots D(p)] \tag{8}$$

The proposed charts are based on Daubechies wavelet analysis, which includes making two charts: the first for controlling the MCUSUM Daubechies approximation coefficients and the second for controlling the MCUSUM Daubechies detail coefficients.

Step 3. Converting coefficient matrices (XA and XD) to standard data through the following:

$$\overline{XA} = [\bar{A}(1)\ \bar{A}(2) \dots \bar{A}(p)] \tag{9}$$

$$\overline{XD} = [\bar{D}(1)\ \bar{D}(2) \dots \bar{D}(p)] \tag{10}$$

$\overline{XA}$ represents the approximation coefficients, mean vector, and $\overline{XD}$ represents the detailed coefficients' mean vector. Estimation of variance and covariance matrix for standard data for approximation and detail coefficients (CA and CD):

$$CA = \begin{bmatrix} V(XA(1,1)) & COV(XA(1,2)) & \dots & COV(XA(1,p)) \\ COV(XA(2,1)) & V(XA(2,2)) & \dots & COV(XA(2,p)) \\ \vdots & \vdots & \ddots & \vdots \\ COV(XA(p,1)) & COV(XA(1,2)) & \dots & V(XA(p,p)) \end{bmatrix} \tag{11}$$

$$CD = \begin{bmatrix} V(XD(1,1)) & COV(XD(1,2)) & \dots & COV(XD(1,p)) \\ COV(XD(2,1)) & V(XD(2,2)) & \dots & COV(XD(2,p)) \\ \vdots & \vdots & \ddots & \vdots \\ COV(XD(p,1)) & COV(XD(1,2)) & \dots & V(XD(p,p)) \end{bmatrix} \tag{12}$$

$$ZA = CA^{-1/2} \times [A(t,:)' - \overline{XA}] \quad for \ t = 2, 3, \dots, n \tag{13}$$

$$\mathrm{ZD} = \mathrm{CD}^{-1/2} \times [\mathrm{D}(t,:)' - \overline{\mathrm{XD}}] \quad for\ t = 2, 3, \dots, n \tag{14}$$

ZA represents the standard deviation vector for approximation coefficients, and ZD represents the standard deviation vector for detail coefficients.

Step 4. Calculate the MCUSUM statistic for the approximation and detail coefficients, which are the points drawn on the proposed charts at optimal (k), as follows:

$$\mathrm{SA}(t) = \max(0, \quad \mathrm{SA}(t-1) + norm(\mathrm{ZA}) - Optimal(k) \tag{15}$$

$$\mathrm{SD}(t) = \max(0, \quad \mathrm{SD}(t-1) + norm(\mathrm{ZD}) - Optimal(k) \tag{16}$$

For $t = 2,3, \dots, n$. SA(t) represents the MCUSUM Daubechies wavelet approximate coefficients (DWAC) statistic plotted on the MCUSUM DWAC chart. SD(t) represents the MCUSUM Daubechies wavelet detail coefficients (DWDC) statistic plotted on the MCUSUM DWDC chart.
The optimal reference value k based on an MCUSUM chart simulation of a set of values (k = 0.1, 0.2, …, 2.5) from a multivariate normal distribution (p = 2, 3 4) for sample size (10000) and iteration (5000) for target ARL (200) with control limit h = 3, 4, …, 8.

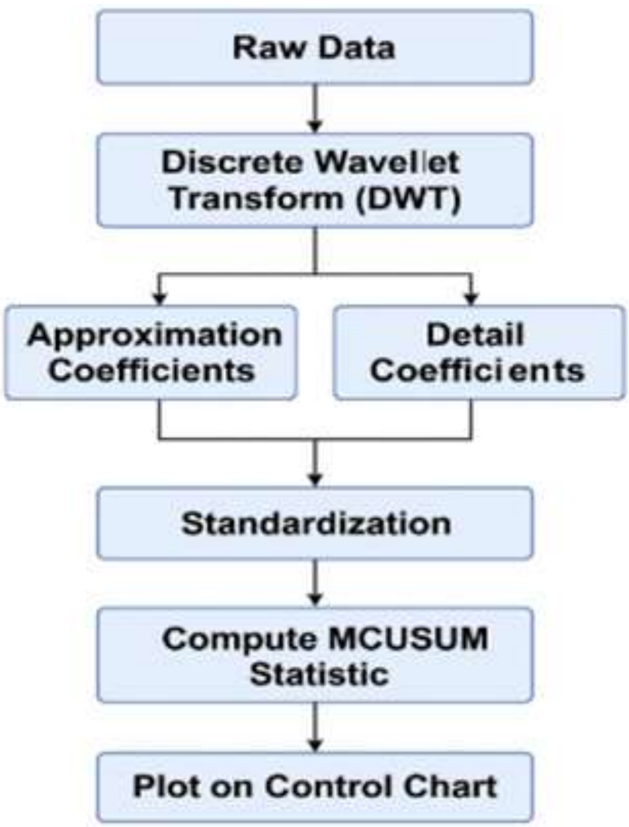

Figure 1: Process flow diagram of the proposed MCUSUM control chart procedure.

As illustrated in Figure 1, the proposed methodology involves applying a Discrete Wavelet Transform (DWT) to raw data, extracting coefficients, standardizing them, and computing the MCUSUM statistic for monitoring.

3 Results and Discussions

Depending on simulation studies and real production data, the proposed wavelet-based MCUSUM control charts were evaluated for their efficiency in detecting process shifts and handling outliers, and their performance was compared with traditional multivariate Shewhart control charts.

3. 1 Simulation Study

To illustrate the idea of the proposed charts and the points drawn on them, multivariate normal distribution data were generated for a zero mean vector and a variance and covariance matrix using the optimal reference value k based on an MCUSUM chart simulation of a set of values (k = 0.1, 0.2, …, 2.5) and ARL (200) with control limit h = 3, 4, …, 8. Sample size (10000) and generation iteration (5000) were used to obtain the optimal reference value k and used in classical and proposed charts to detect out-of-control signals that were out of control for sample size (100) for a different number of variables (p = 2, 3, and 4).
wavelet analysis was performed to calculate the approximation and detail coefficients of the Db (40) wavelet and summarize the results in Table 1 and shown in Figure 2:

Table 1: Simulate two observations and DWT coefficients.

X_{j1}	X_{j2}	A	D
5.7604	7.5935	9.4426	-1.2962
7.4894	12.2559	13.9621	-3.3705
15.1002	14.6115	21.0094	0.3455
18.8942	20.9893	28.2019	-1.4815

Table 1 shows two observations (Xj_1, Xj_2) for four variables and their corresponding wavelet approximation (A) and detail (D) coefficients derived from the discrete wavelet transform (DWT). This data is used to compute MCUSUM statistic, which monitors deviations from the control, thus identifying when the process is out of control. For instance, the table includes values like 9.4426 for approximation coefficients (low pass filter) and -1.2962 for detail coefficients (high pass filter). These coefficients are key inputs to the MCUSUM charts, helping to track both general trends and finer variations in the data.
The wavelet analysis of two observations produces two coefficients, one of which represents the approximation and is proportional to the average of the two observations, and the other represents the detail and is proportional to the difference between the two observations, and so on for the other variables. DWT coefficients for the first experiment simulation of two variables and 100 observations are shown in Figure 3.

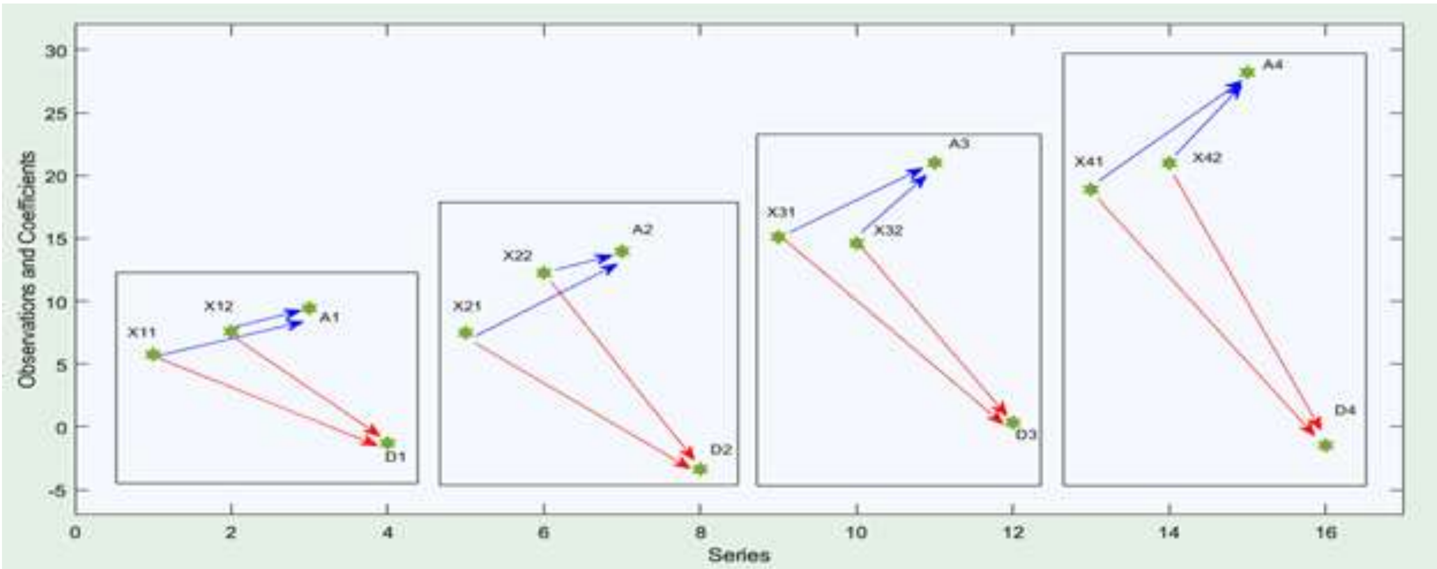

Figure 2: Wavelet analysis for two observations and four Variables.

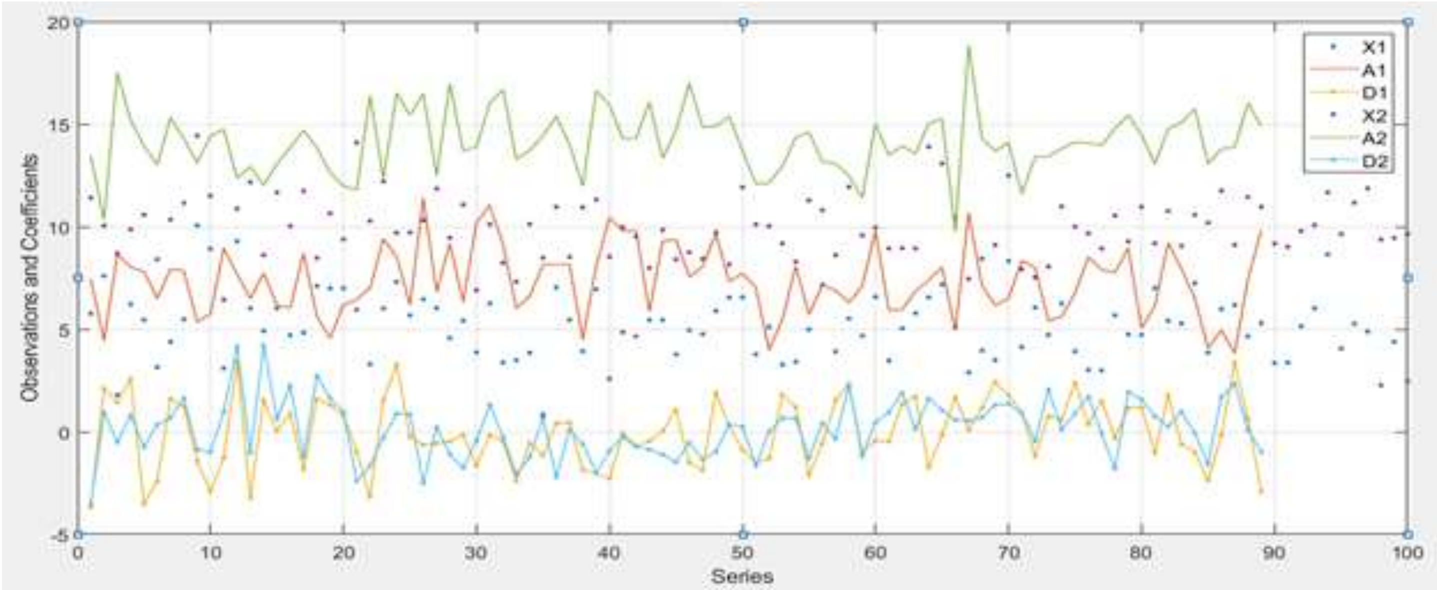

Figure 3: Wavelet analysis for (100) observations and two Variables.

Simulation MCUSUM, MCUSUM-DWAC, and MCUSUM-DWDC charts for the set of values (k = 0.1, 0.2, …, 2.5) and ARL (200) with control limit h = 3, 4, …, 8. Sample size (10000) and generation iteration (5000) were used to obtain the optimal reference value k and used in classical and proposed charts to out-of-control signals that were out of control for sample size (100) for a different number of variables (p = 2, 3, and 4) as in Tables 2-4.

In Table 2, the classical MCUSUM chart generates different ARLs for two variables with differing control limits (h) and ideal reference values (k). For control limits h = 3 to h = 8, the MCUSUM-DWAC chart continuously shows an increasing number of out-of-control signals. It's interesting to note that, except for h = 3, the MCUSUM-DWDC chart is more sensitive and indicates out-of-control conditions at practically every h level. This suggests that detail coefficients (DWDC) are superior to approximation coefficients (DWAC) in identifying minute changes. MCUSUM-DWAC and MCUSUM-DWDC charts provide a robust system where DWAC captures broader trends for low-pass filters and DWDC focuses on more abrupt changes for high-pass filters. Adjusting h = 7 and k = 1.3 optimally can provide a balanced detection mechanism and greater ARL (260.264).

Table 2: Target ARL (200) for two variables (Simulation).

Chart	*h*	The optimal reference value (*k*)	ARL	Out-of-control signal
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	3	1.5	167.792	11 0 14
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	4	1.4	179.651	12 32 14
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	5	1.3	120.871	12 31 14
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	6	1.3	181.283	21 46 23
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	**7**	**1.3**	**260.264**	**21** **0** **24**
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	8	1.2	100.005	21 44 22

Table 3: Target ARL (200) for three variables (Simulation).

Chart	*h*	The optimal reference value (*k*)	ARL	Out-of-control signal
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	**3**	**1.9**	**246.325**	**12** **0** **0**
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	4	1.7	113.417	12 0 0
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	5	1.7	201.915	22 0 0
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	6	1.6	106.323	22 0 0
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	7	1.6	140.778	23 0 0
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	8	1.6	177.056	0 0 0

In Table 3, the case of three variables, the classical MCUSUM chart shows inconsistent ARL values, with no significant out-of-control signals on either the MCUSUM-DWAC or MCUSUM-DWDC charts across all control limits. This implies that both wavelet-based charts may not provide additional detection benefits for the three variables under the given simulation parameters.

MCUSUM-DWAC and MCUSUM-DWDC charts provide a robust system where DWAC captures broader trends for low-pass filters and DWDC focuses on more abrupt changes for high-pass filters. Adjusting h = 3 and k = 1.9 optimally can provide a balanced detection mechanism and greater ARL (246.325). The MCUSUM-DWAC and MCUSUM-DWDC charts register out-of-control signals across the different parameter settings in this simulation. This may indicate that the changes in the process are not significant enough in either the low-pass or high-pass filters to trigger these charts.

Table 4: Target ARL (200) for four variables (Simulation).

Chart	h	The optimal reference value (k)	ARL	Out-of-control signal
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	3	2.1	125.864	12 89 0
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	4	2	124.611	22 0 0
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	**5**	**2**	**235.559**	**0** **0** **0**
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	6	1.9	120.955	39 0 0
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	7	1.9	163.328	0 0 0
MCUSUM MCUSUM-DWAC MCUSUM-DWDC	8	1.9	217.012	0 0 0

For four variables, the classical MCUSUM chart still performs well in terms of ARL, but the MCUSUM-DWAC chart detects a high number of out-of-control signals for h = 3, while no signals are detected by either the MCUSUM-DWDC or DWAC charts for most other control limits. This suggests that, for higher-dimensional datasets, the approximation coefficients (DWAC) are potentially more useful for identifying large shifts. At h = 5 and k = 2 optimally can provide an optimal detection mechanism and greater ARL (235.559). The MCUSUM-DWAC and MCUSUM-DWDC charts register out-of-control signals across the different parameter settings in this simulation. This may indicate that the changes in the process are not significant enough in either the low-pass or high-pass filters to trigger these charts.

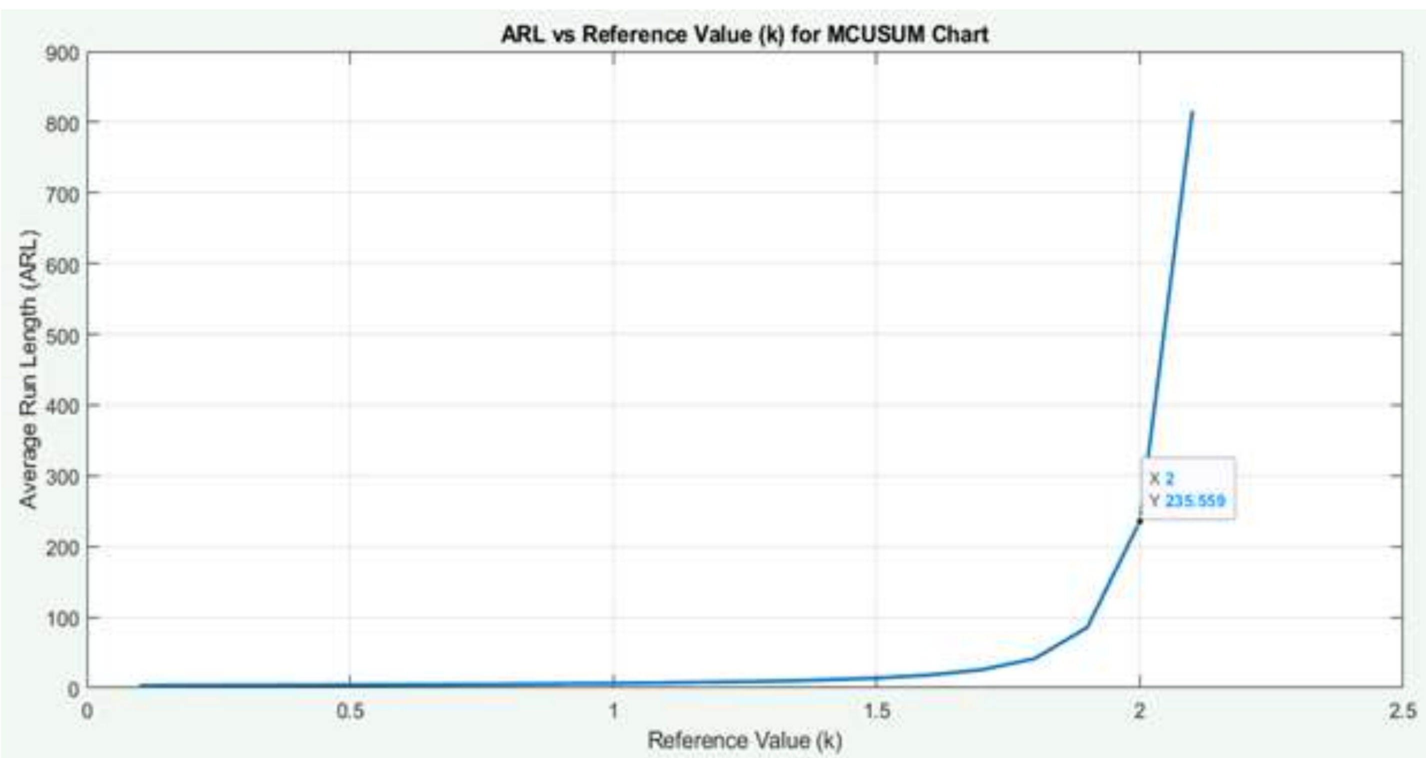

Figure 4: ARL for the MCUSUM chart and optimal reference value k.

Figure 4 displays the relationship between the Average Run Length (ARL) and the reference value k for an MCUSUM chart. The optimal reference value for the MCUSUM chart is approximately k = 2, where the ARL is 235.559. Beyond this value, the ARL increases sharply, reducing the chart's efficiency in detecting process changes. Keeping k close to 2 ensures a balanced detection capability without overly long delays in signaling shifts. CUSUM statistics for the two variables (first experiment simulation) of the MCUSUM, MCUSUM-DWAC, and MCUSUM-DWDC charts were calculated in Figures 5-7. To control and monitor the quality of the product, the results were drawn on these charts for the value of h and k, and optimal ARL.

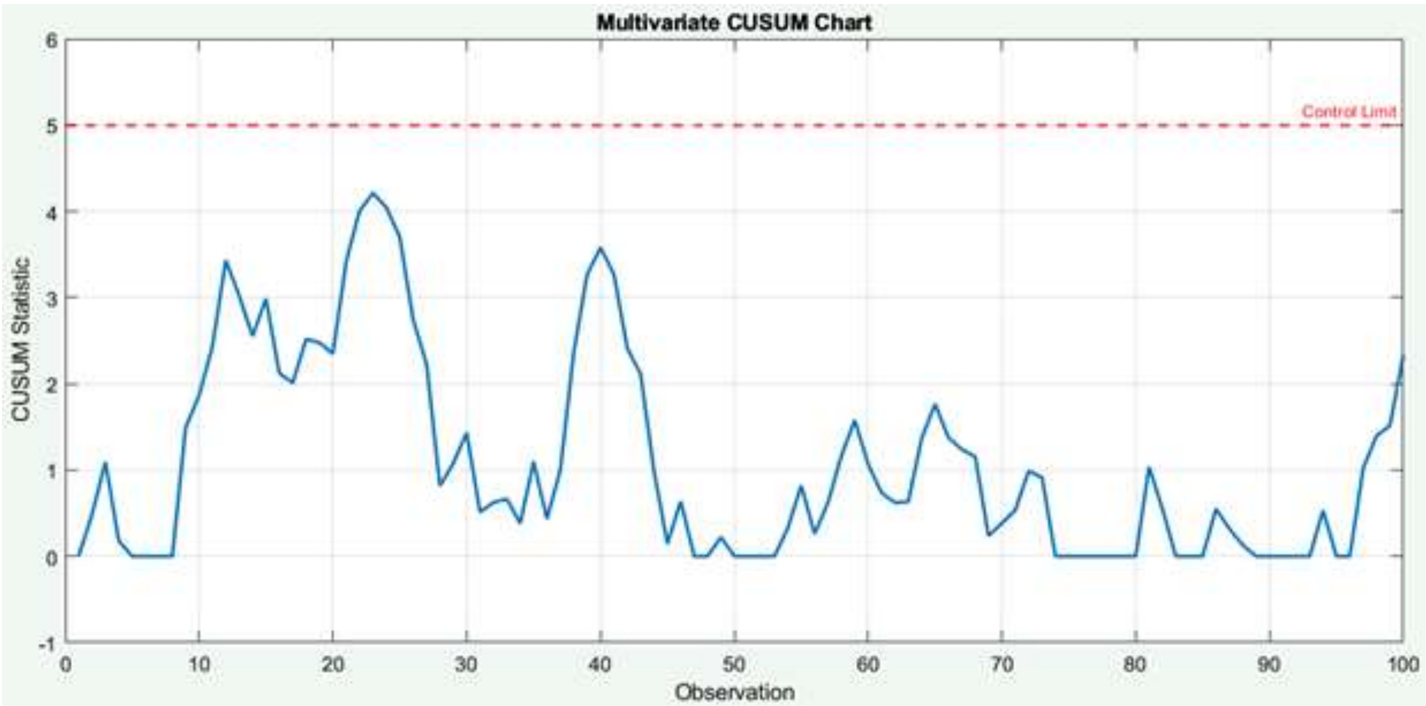

Figure 5: MCUSUM chart for the Simulation.

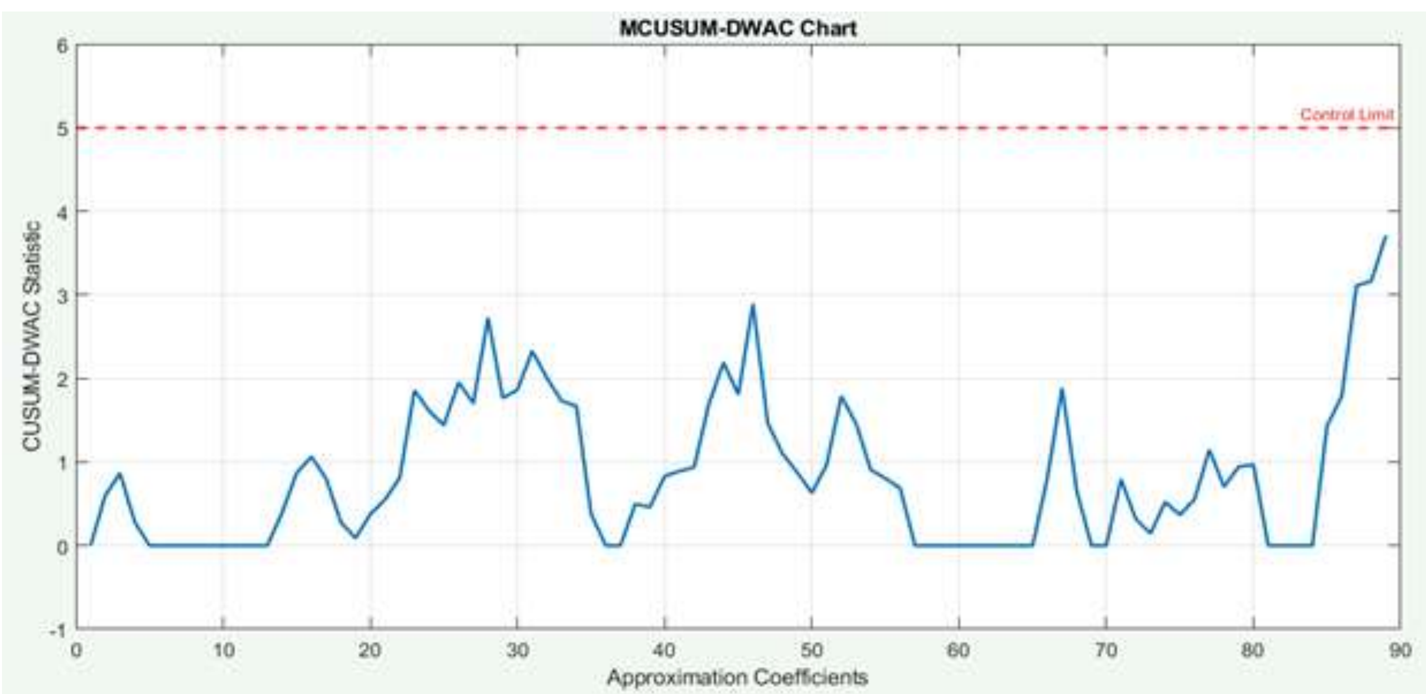

Figure 6: MCUSUM-DWAC chart for the Simulation.

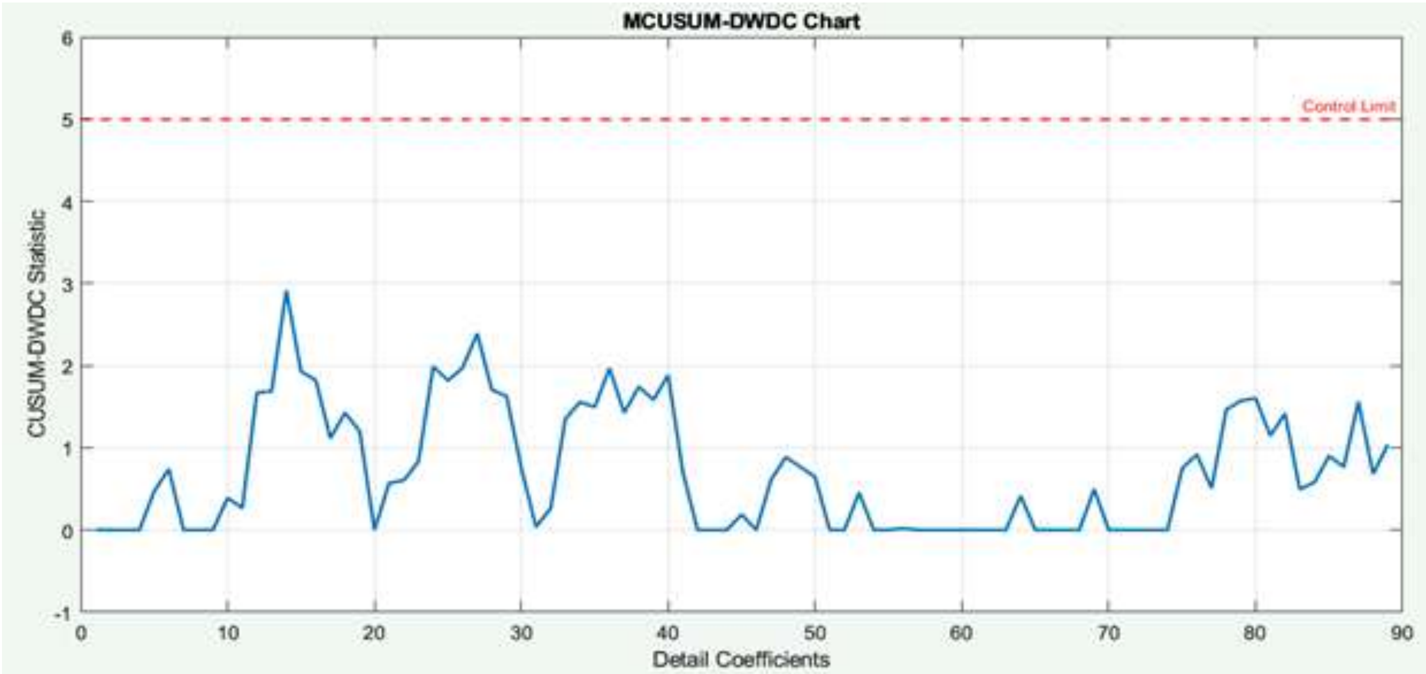

Figure 7: MCUSUM-DWDC chart for the Simulation.

3. 2 Real Data

The source of this real data is Erbil Steel company, which is a construction company commencing in 2006 in Northern Iraq. Erbil Steel Company started integrated steel production in December 2007. The facility, which produces its energy through its 32 MW powerhouse, has an annual steel production capacity of 240,000 Tons. This data concluded 100 observations on a Nominal diameter of 12 mm, which is composed of 2 different variables: Yield point and Tensile strength.

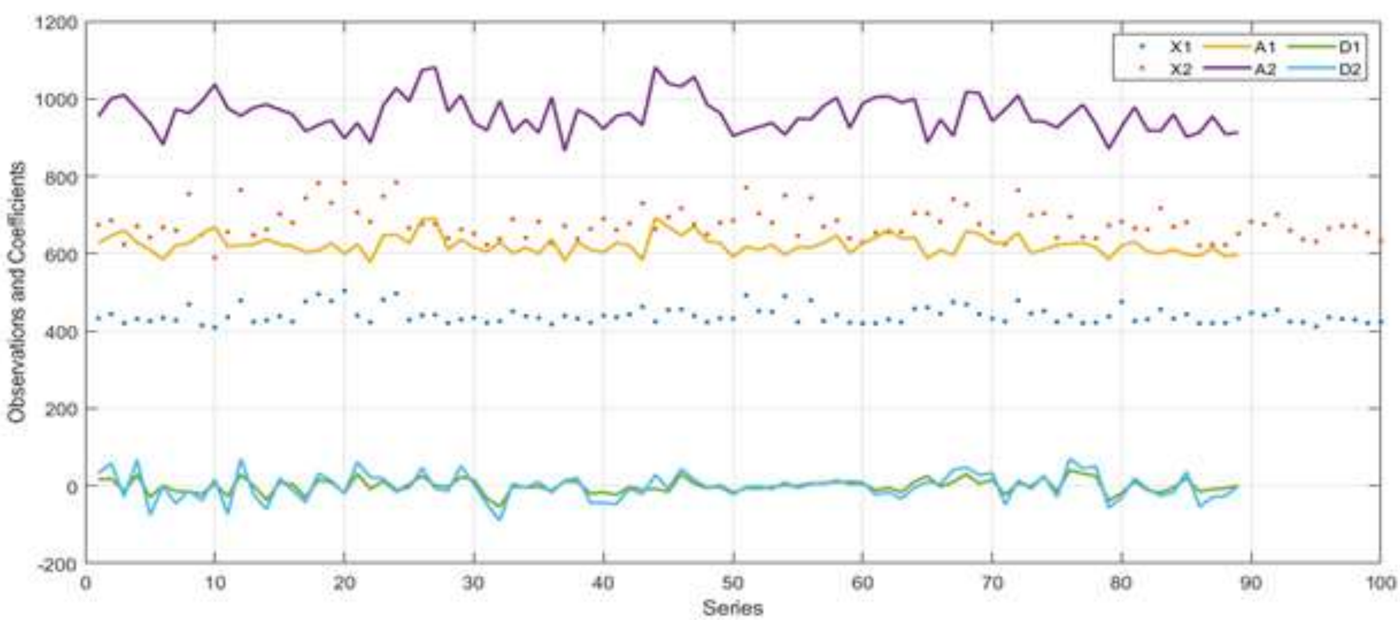

Figure 8: Wavelet analysis for real data.

Figure 8 presents original data (X1, X2), approximation coefficients (A1, A2), and detail coefficients (D1, D2) obtained from wavelet transformation. The original values appear as scattered points, while approximation coefficients show the overall trend, and detail coefficients reflect fluctuations around zero. CUSUM statistics for the two variables, Yield point and Tensile strength of MCUSUM, MCUSUM-DWAC, and MCUSUM-DWDC charts were calculated. To control and monitor the quality of the Erbil Steel product, the results were drawn on these charts and summarized according to the value of *h* and *k*, and optimal ARL in Table 5:

Table 5: Control and monitor the quality of the Erbil Steel product.

Chart	*h*	The optimal reference value (*k*)	ARL	Out-of-control signal
MCUSUM				18
MCUSUM-DWAC	3	1.5	167.792	27
MCUSUM-DWDC				0
MCUSUM				18
MCUSUM-DWAC	4	1.4	179.651	27
MCUSUM-DWDC				0
MCUSUM				18
MCUSUM-DWAC	5	1.3	120.871	27
MCUSUM-DWDC				32
MCUSUM				20
MCUSUM-DWAC	6	1.3	181.283	28
MCUSUM-DWDC				32
MCUSUM				**20**
MCUSUM-DWAC	**7**	**1.3**	**260.264**	**46**
MCUSUM-DWDC				**0**
MCUSUM				20
MCUSUM-DWAC	8	1.2	100.005	44
MCUSUM-DWDC				32

CUSUM Chart: At *h* = 5 and *k* = 1.3 the ARL is 120.871 which indicates that the chart can detect changes faster compared to *h* = 6 and ARL = 181.283 and the observation sequence (18) that was out of control at different *h* values 3, 4, and 5 indicates a reliable detection mechanism without excessive false alarms, it also results in a balance between maintaining high sensitivity (detecting small to moderate changes quickly) and avoiding false alarms. In quality control applications, rapid detection of transitions is essential. The choice of h = 5 reflects a practical approach to ensure that large changes in the process can be detected on time, without the system becoming overly sensitive and triggering unnecessary alarms. CUSUM-DWAC Chart: At *h* = 7 and *k* = 1.3, the observation sequence (46) was outside the control limits, indicating that the chart is effective at detecting only large changes while filtering out noise. This higher control limit allows for a longer ARL (260.264), providing more stable operation over time in environments where small fluctuations can be common. With a focus on low-frequency changes, a higher *h* value helps ignore minor deviations, which is critical in stable operations where the focus is on identifying large changes rather than responding to every small fluctuation. Practical application: In scenarios such as manufacturing or industrial processes, where noise can obscure meaningful signals, a higher *h* reduces false alarms and ensures that only the most relevant and largest shifts trigger a response. Thus, *h* = 7 is ideal for long-term monitoring, providing a buffer against noise. CUSUM-DWDC Chart: At *h* = 5 and *k* = 1.3, the observation sequence (32) was outside the control limits; it indicates sensitivity but still maintains a reasonable response to changes that can occur in the production process. The ARL (120.871) associated with this configuration allows for immediate detection without an excessive number of false alarms. Like the MCUSUM chart, *h* = 5 in MCUSUM-DWDC ensures high sensitivity to sudden high-frequency changes. This is critical for applications that require immediate attention to subtle changes that can occur in the production process, such as critical industrial processes. Through the results, these charts will be used in Phase 2 to control and monitor the Erbil Steel product for new data representing 60 values of Yield point and Tensile strength (Table 6).

Table 6: Control and monitor the quality of the Erbil Steel product (Phase 2).

Chart	*h*	The optimal reference value (*k*)	ARL	Out-of-control signal
MCUSUM	5		120.871	8
MCUSUM-DWAC	7	1.3	260.264	37
MCUSUM-DWDC	5		120.871	0

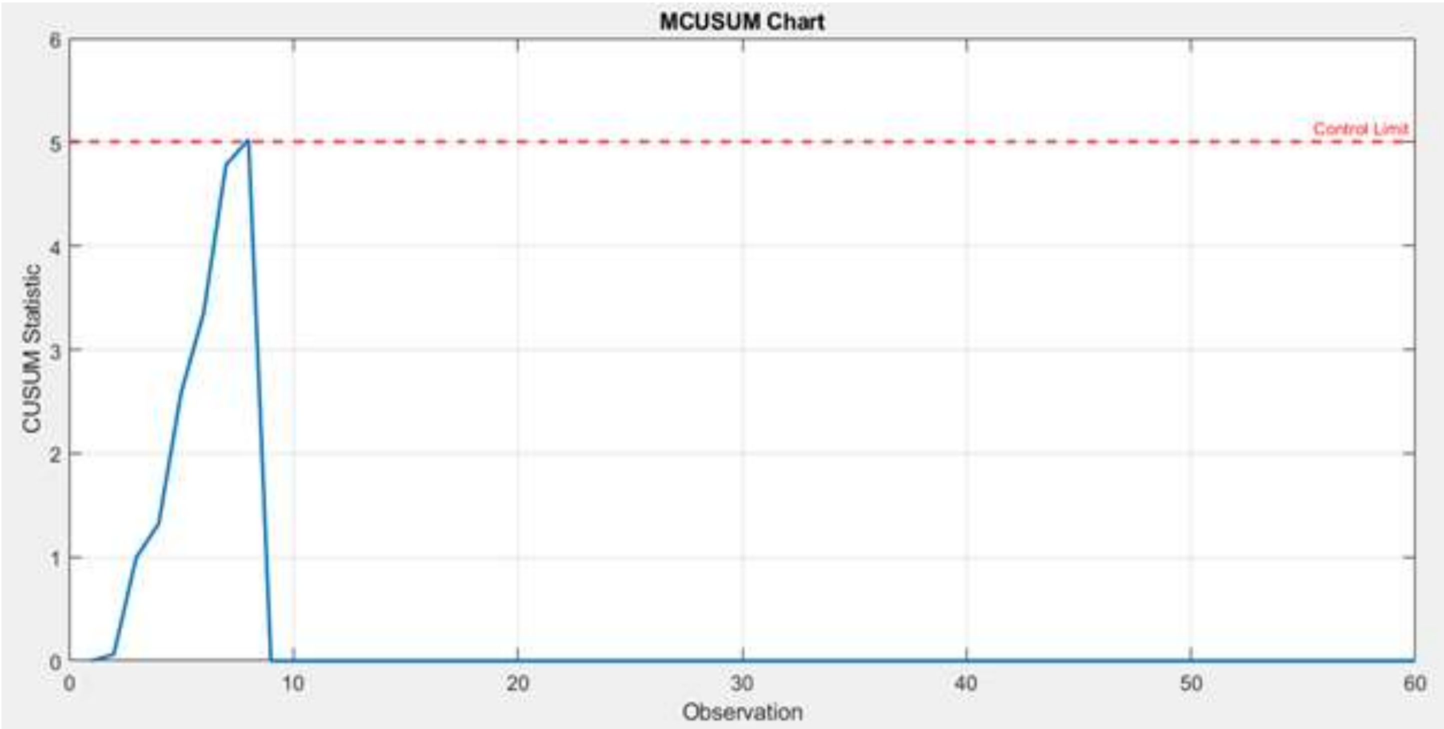

Figure 9: MCUSUM chart for the Erbil Steel product quality (Phase 2).

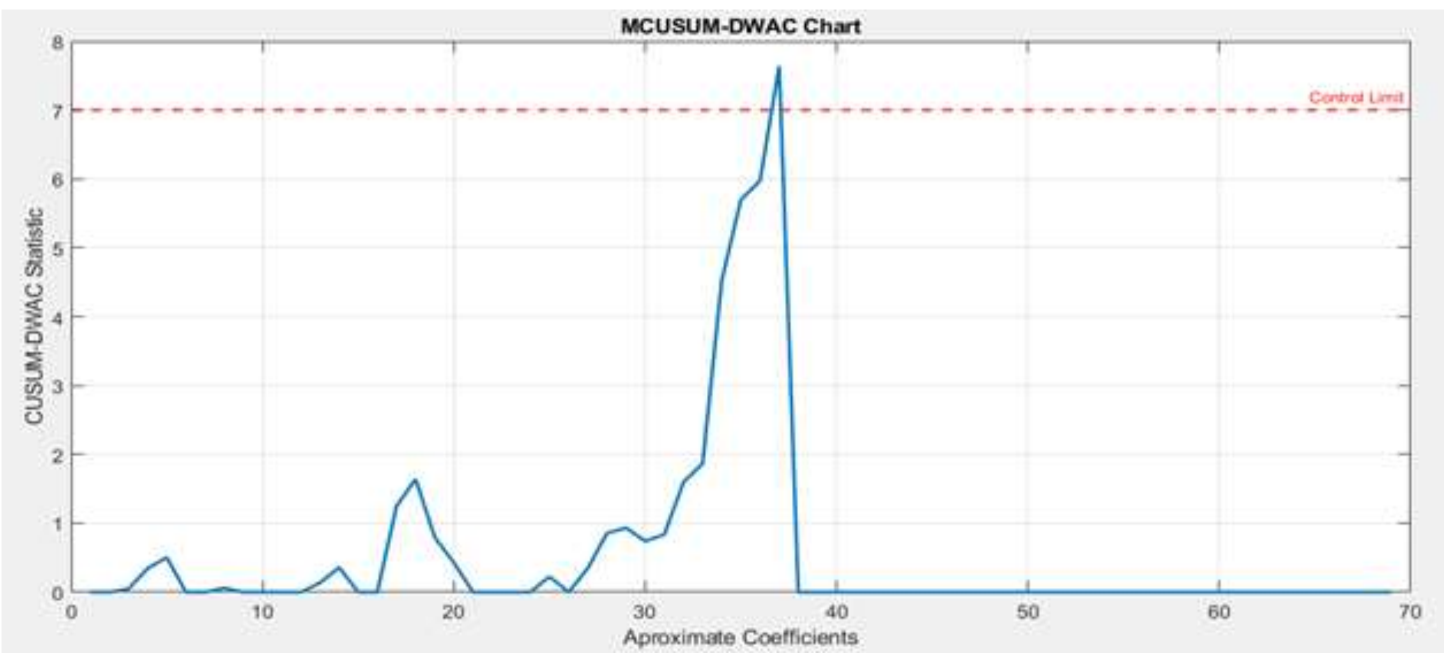

Figure 10: MCUSUM-DWAC chart for the Erbil Steel product quality (Phase 2).

Figures 9-11 show three charts in Phase 2. The MCUSUM chart has 8 out-of-control signals, the MCUSUM-DWAC chart shows 37 out-of-control signals, indicating it may be more sensitive to certain types of shifts in the process, and the MCUSUM-DWDC chart has no out-of-control signals (0), suggesting that either the process remains stable for this method, or it is less sensitive to the variations in this dataset. The MCUSUM chart provides balanced sensitivity to shifts in the process mean (ARL = 120.871). With an out-of-control signal count of 8, it's useful for moderate control over yield point and tensile strength. The MCUSUM-DWAC chart has a high ARL of 260.264 and a greater number of out-of-control signals (37), suggesting it may be more conservative but sensitive to even minor deviations. The MCUSUM-DWDC chart with no out-of-control signals shows the most stability, but this may indicate lower sensitivity if the chart does not detect known shifts. Depending on the control priorities, each chart offers different levels of sensitivity. If immediate response to even small shifts is crucial, the MCUSUM-DWAC chart might be more appropriate. However, if fewer interventions and longer runs without interruptions are desired, the MCUSUM-DWDC chart may be a better option. Finally, it can be said that there is a problem with the Erbil Steel product quality, and the engineering department must review the production line to identify and address the defect.

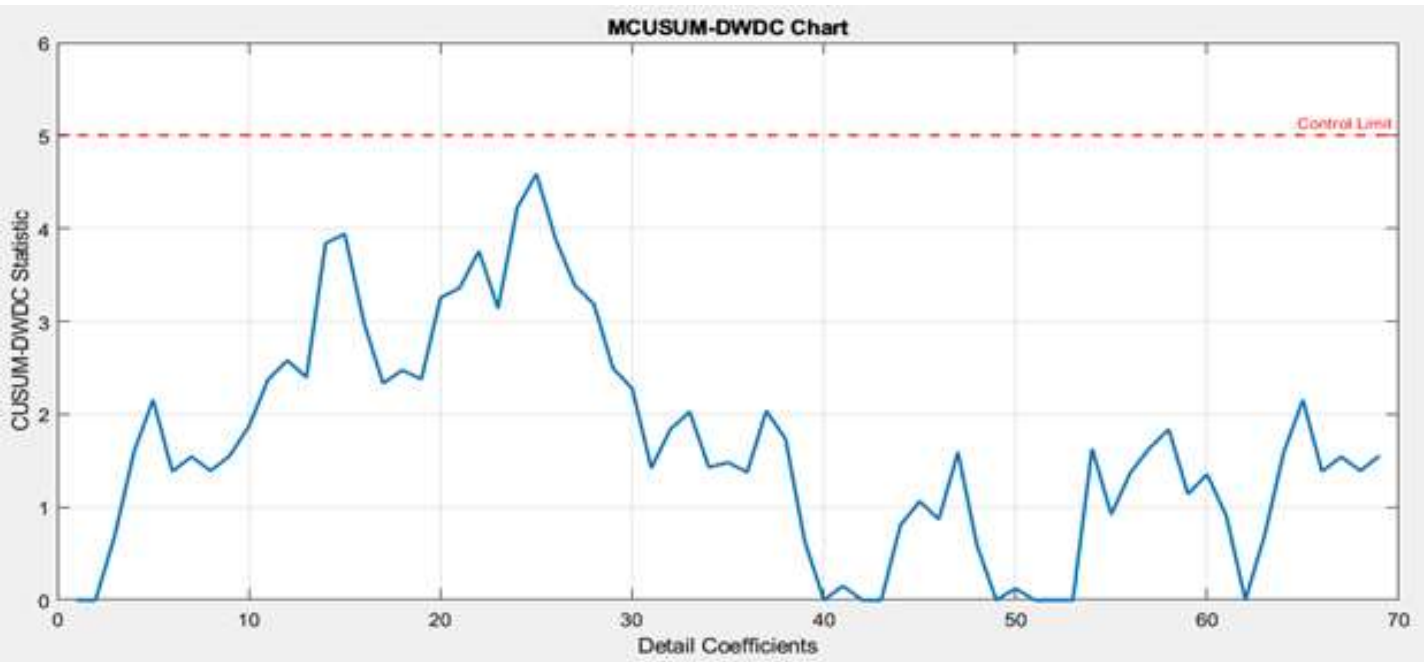

Figure 11: MCUSUM-DWDC chart for the Erbil Steel product quality (Phase 2).

4 Conclusions

1. This study successfully developed multivariate CUSUM charts that utilize wavelet analysis, specifically the discrete wavelet transforms of the Daubechies wavelet of order 40.
2. Using both approximation and detail coefficients. Two separate charts were designed for monitoring and controlling different quantitative characteristics in production processes. The first chart targets the averages of these characteristics, while the second focuses on their standard deviations.
3. Employing Mahalanobis distance as the foundation for the cumulative sum statistics has yielded promising results, enhancing the sensitivity to detect shifts in process quality.
4. The proposed MCUSUM Daubechies wavelet charts provide a robust quality control method and show superior performance to traditional control charts, especially regarding efficiency while maintaining the average run length.

5 Recommendations

1. Evaluate the use of Discrete Wavelet Transform (DWT) coefficients of Coiflets wavelets in innovating MCUSUM control charts as an alternative to Daubechies wavelets.
2. Check the application of Coiflets wavelet transform to obtain approximation and detail coefficients for supervising outcome steps
3. Utilize approximation coefficients to make charts for controlling the means of qualitative characteristics and detail coefficients for controlling variability or standard deviation.
4. Evaluate whether Coiflets wavelets offer innovative sensitivity and robustness in comparison to Daubechies wavelets.
5. Weigh the performance of Coiflets-based MCUSUM charts over different industries, especially in fields where signal symmetry and flawlessness are important.
6. Find out optimal reference values for control limits and their impact on the average run length (ARL) to escalate quality control methodologies.

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