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Original Article

Binary Soft Fuzzy approximation space

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ABSTRACT

In this work devoted to study Binary soft fuzzy probability, probabilistic interpretation of Binary soft fuzzy, relationship between BSF reliability and classical reliability in addition to that BSF reliability is presented in view of BSF approximation, consider reliability as probability according to definition and consider BSF in the result according to our belief in current study.

Keywords: BSF-reliability, BSF-approximation space, BSF lower approximation and BSF upper approximation.

1 Introduction

Sangoor A.A[16] binary soft fuzzy set. The first research on fuzzy fault tree analysis was introduced by Tanaka et al in (1983) the treated probability of basic event as trapezoidal fuzzy number to simply the calculation Cetkin, V.,[2]. A note on fuzzy topological spaces, cheng [3] Fuzzy topological spaces, considered the failure probability of basic event as triangular fuzzy number. Feng, [5]. Soft sets and soft rough sets. Al-Swidi, L. A. A [1] Soft fuzzy and fuzzy soft fuzzy theory with applications, Varol, B. P [23] Fuzzy soft topology. Maji [8]) used to calculate the reliability of different models of systems. Ming, P. P [9] Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith convergence, Molodtsov [10] introduced the notion of soft set and started to develop the basic of the corresponding theory as a new approach for modeling uncertainties. Mostafa, [11] Soft B-rough sets and their application to determine COVID-19, Pawlak [13] Rough sets, Roy [14] A note on fuzzy soft topological spaces. Saber [15] Ideal on fuzzy topological spaces. Serkan, A., kalpana [19,7] On fuzzy soft topological spaces. Shabir and Naz, [20] On soft topological spaces, Topological structures of fuzzy soft sets. Tany, B., & Kandemir, M. B[22] Topological structures of fuzzy soft sets. The concept of fuzzy set introduced by Zadeh [24], Sarhad F. Namiq, Ennis Rosas [17] On $\lambda^D - R_0$ and $\lambda^D - R_1$ spaces. Sarhad F. Namiq, Ennis Rosas [18], New types of connected component spaces and maximal open set via s-operation. Sostak, A. P. On fuzzy topological structure [21]. Halgwrd M. Darwesh, Sarhad F. Namiq [6] pre-irresolute functions in closure spaces. P. Gino Metilda and J. Subhashini [12], Remarks on fuzzy binary soft limit points. E. Rosas, C. Carpintero, J. Sanabria [4], Characterization of I-b-open sets.The main objective of this work is, therefore, to present a new type of soft fuzzy approximate called Binary Soft Fuzzy approximate. First of all, let us define the Binary soft fuzzy approximate as follows. In the later part of this paper, we will also explain the Binary soft fuzzy approximate and look at the primary and essential operations on this set with examples.

The purpose of the present paper is to introduce a new type of soft fuzzy approximate that contains the concept of a binary on many sets of parameters that can be applied in one of the essential scientific disciplines in the future.

This article is grouped into sections, as outlined below. Section introduces our new idea, mentions some central concepts Binary soft fuzzy probability, probabilistic interpretation of Binary soft fuzzy, relationship between BSF reliability and classical reliability. In we present a new definition of the soft fuzzy approximation space. we also illustrate the essential and central operations for binary soft fuzzy approximation space.

2 Preliminaries

In this section, we view the important notions of approximation space, lower approximation and upper approximation space, the boundary, positive, negative region.

2. 1 Definition [10]:

Let μ be a universe finite set and R be an equivalence relation on μ. μ /R = {$[x]_R$: x ∈ μ }will denote to the family of equivalence classes of R on μ . Then, the pair (μ, R) is said to be an approximation space. The lower and upper approximation of X ⊆ μ are defined respectively by

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$\underline{R(X)}$ = {x ∈ μ : $[x]_R$ ⊆ X}, and $\overline{R(X)}$= {x ∈ μ : $[x]_R$ ∩ X ≠ ∅}. According to Pawlak's [11] definition, X is called a rough set if $\underline{R(X)} \neq \overline{R(X)}$.

2. 2 Definition [10]:

Let (μ, R) be an approximation space. Then, the boundary, positive, negative regions and accuracy for the approximations of X ⊆ μ are defined respectively by $BND_R(X) = \overline{R(X)} - \underline{R(X)}$

$POS_R(X) = \underline{R(X)}$,

$NEG_R(X) = \mu - \overline{R(X)}$, and

$\mu_R(X) = \left|\frac{\underline{R(X)}}{\overline{R(X)}}\right|$

, where $|\overline{R(X)}| \neq 0$.

Obviously, $\mu_R(X) \leq 1$. If $\mu_R(X) = 1$, then X is called exact. Otherwise, it is called rough.

2. 3 Definition [3]:

Let S = (F, £) be a soft set over μ. Then, a pair A_s = (μ, S) is called a soft approximation space. The soft As -lower approximation and soft As -upper approximation for X ⊆ μ are defined respectively by

$\underline{S(X)}$= {u ∈ μ: ∃e ∈ £, [u ∈ F (e), F (e) ∩ X≠ ∅]} The soft As -lower approximation,

$\overline{S(X)}$ = {u ∈ μ: ∃e ∈ £, [u ∈ F (e) ⊆ X]}, and soft As -upper approximation,

In general, $\overline{S(X)}$ and $\underline{S(X)}$ are referred to soft rough approximations with respect to A_s. Moreover

$POSA_s(X) = \underline{S(X)}$

$NEGA_s(X) = \mu - \overline{S(X)}$, and

$BNDA_s(X) = \overline{S(X)} - \underline{S(X)}$ are called soft A_s -positive region, soft A_s -negative region and soft A_s -boundary region of X. If $\overline{S(X)} = \underline{S(X)}$ means $BNDA_s(X) = \emptyset$, then X is said to be soft As -definable or soft As -exact set. Otherwise, X is soft A_s -rough. Moreover,

$\mu A_s(X)| = \left|\frac{\underline{S(X)}}{\overline{S(X)}}\right|$, where $|A_s(X)|\neq 0$. It is called soft A_s -accuracy of X.

2. 4 Proposition [10]:

Let S = (F, £) be a full soft set over μ and A_s = (μ, S) be a soft approximation space. Then the conditions are held:

(i) $\underline{S(\mu)} = \overline{S(\mu)} = \mu$. (iii) $\overline{S}(\{x\}) \neq \emptyset \ \forall \ x \in \mu$,

(ii) X ⊆ $\overline{S(X)}$,∀ X.

2. 5 Definition [1]:

Let μ be an initial universe, $£_1, £_2$ are two sets of parameters, $\mathcal{P}(\mu)$ represents μ's power set. A binary soft fuzzy set (bsfs-set) ψ on μ with member-ship mapping f_ψ representing by a combination between binary- soft set and a member-ship mapping of fuzzy set with respect to power set $f: \mu \to \mathbb{I} = [0,1]$; together with binary-soft set $S: £_1 \times £_2 \to \mathcal{P}(\mu)$, Then $f_\psi: £_1 \times £_2 \to l_m^7$ (the set of all fuzzy sets in seventh family), is a member-ship mapping of BSF-set ψ where;

$\psi = \left\{\left(e_i, e_j\right), f_\psi\left(e_i, e_j\right)\right\}, \forall\left(e_i, e_j\right) \in £_1 \times £_2$, where $e_i \in £_1$ and $e_j \in £_2$, $f: £_1 \times £_2 \to l_m^7$, i = 1, ... , v ; j = 1, ... , u }.

3 BSF approximation space

In this section, Binary soft fuzzy probability, probabilistic interpretation of Binary soft fuzzy, relationship between BSF reliability and classical reliability depend on [9].

3. 1 Definition:

let be $(\psi, £_1 \times £_2)$ be BSF over μ . then a pair (μ, ψ) is BSF approximation space. The A_{BSF} -BSF lower approximation space and the A_{BSF} -BSF upper approximation space for $N \,\tilde{\subseteq}\, \mu$ and defined as following:

$\underline{\psi}$ (N)=$\sqcup_{(e_i, e_j)}\{ \times\in \mu, \exists\, e_i, e_j \in £_1 \times £_2, [\times\in \psi \,\tilde{\subseteq}\, N]\}$,

$\bar{\psi}$ (N)=$\sqcup_{(e_i, e_j)}\{ \times\in \mu, \exists\, e_i, e_j \in £_1 \times £_2, [\times\in \psi \cap N \neq \bar{\bar{0}}]\}$, such that

$\underline{\psi}, \bar{\psi}$ are BSF rough approximation space with respect to A_{BSF}.

3. 2 Example:

let be $£ = \{e_1, e_2, e_3, e_4\}$, $£_1 = \{e_1, e_2\}$, $£_2 = \{e_3, e_4\}$, let μ be universal set consisting of $\mu = \{\times_1, \times_2, \times_3, \times_4\}$, $N = \{\times_1, \times_2, \times_3\} \,\tilde{\subseteq}\, \mu, \psi: p(\mu) \to l_7^m$, for example 3.1 in [1],

ψ= {(e_1, e_3), {($\times_1$, 0.1), ($\times_2$, 0), ($\times_3$, 0.3), ($\times_4$, 0)}}, ((e_1, e_4), {($\times_1$, 0.1), ($\times_2$, 0), ($\times_3$, 0), ($\times_4$, 0.4)}}),

((e_2, e_3), {($\times_1$, 0), ($\times_2$, 0.2), ($\times_3$, 0.3), ($\times_4$, 0)}}, ((e_2, e_4), {($\times_1$, 0), ($\times_2$, 0.2), (($\times_3$, 0), ($\times_4$, 0.4)}, to find lower and upper BSF approximation space.to find lower and upper BSF approximation space.

$\underline{\psi}$ (N)=$\sqcup_{(}e_i, e_j)\{ \times\in \mu, \exists\, e_i, e_j \in £_1 \times £_2, [\times\in \psi \;\widetilde{\subseteq}\; N]\}$,

$\underline{\psi}$ (N)=$\{ (e_1, e_3), \{(\times_1, 0.1), (\times_2, 0), (\times_3, 0.3)\}, ((e_1, e_4), \{(\times_1, 0.1), (\times_2, 0), (\times_3, 0\}),$

$((e_2, e_3), \{(\times_1, 0), (\times_2, 0.2), (\times_3, 0.3)\}, ((e_2, e_4), \{(\times_1, 0), (\times_2, 0.2), ((\times_3, 0)\},$

$\bar{\psi}$ (N)=$\sqcup_{(}e_i, e_j)\{ \times_i\in \mu, \exists\, e_i, e_j \in £_1 \times £_2, [\times_i\in \psi \cap N \neq \bar{\bar{0}}]\}$,

$\bar{\psi}$ (N)= $\{(e_1, e_3), \{(\times_1, 0.1), (\times_3, 0.3)\}), ((e_1, e_4), \{(\times_1, 0.1)\}),$

$((e_2, e_3), \{(\times_2, 0.2), (\times_3, 0.3)\}, ((e_2, e_4), \{(\times_2, 0.2)\},$

$\bar{A}_{BSF}$ (N)= $\{(e_1, e_3), \{\times_1, \times_3\}), ((e_1, e_4), \{ \times_1\}), ((e_2, e_3), \{(\times_2, \times_3\}, ((e_2, e_4), \{\times_2\},$

$\underline{A}_{BSF}$(N)=$\{ (e_1, e_3), \{\times_1, \times_2, \times_3\}), ((e_1, e_4), \{\times_1, \times_2, \times_3\}),$

$((e_2, e_3), \{\times_1, \times_2, \times_3\}), \; ((e_2, e_4), \{\times_1, \times_2, \times_3\})$. Thus $\bar{A}_{BSF}$ (N)$\neq$ $\underline{A}_{BSF}$(N), so upper to lower- BSF approximation space

$\bar{A}_{BSF}$ ($\underline{A}_{BSF}$(N)) = $\{ (e_1, e_3), \{\times_1, \times_3\}), ((e_1, e_4), \{\times_1\}),$

$((e_2, e_3), \{\times_2, \times_3\}), ((e_2, e_4), \{ \times_2\}) = \bar{A}_{BSF}$ (N)

3. 3 Definition:

Let (μ, ψ) is BSF approximation space. The BSF positive region, BSF negative region and BSF boundary region defined as following:

Pos$\psi(N)$ =$\underline{\psi}$ (N)

Neg $\psi(N) = \mu - \bar{\psi}$ (N) =$(\psi(N))^c$;

Bound $\psi(N) = \bar{\psi}$ (N)- $\underline{\psi}$ (N). are called A_{BSF} -BSF positive region, the A_{BSF} -BSF negative region an A_{BSF} -BSF boundary region of N. $\bar{f_\psi}$ (N)= $\underline{f_\psi}$ (N) if and only if Bound $f_\psi(N) = \bar{\bar{0}}$.

Then N is said to beA_{BSF} -BSF definable, otherwise N is called A_{BSF} BSF rough.

$$\boldsymbol{A_{BSF}} = \begin{cases} = \bar{\bar{0}}, & N\ BSF definable \\ \neq \bar{\bar{0}}, & N\ BSF\ rough \end{cases}$$

Moreover, the BSF accuracy of N defined,

$M_{BSF}(N) = M_\psi(N) = \left|\frac{\underline{\psi}\,(N)}{\bar{\psi}\,(N)}\right|$, where $|\overline{A_{BSF}}| \neq \bar{\bar{0}}$, is called A_{BSF} -BSF-accuracy of N.

3. 4 Example:

Let $£ = \{e_1, e_2, e_3, e_4\}$, $£_1 = \{e_1, e_2\}$, $£_2 = \{e_3, e_4\}$, μ be universal set consisting of $\mu = \{\times_1, \times_2, \times_3, \times_4\}$,N=$\{\times_1, \times_2, \times_3\} \widetilde{\subseteq}\ \mu$,$\psi$: $p(\mu) \rightarrow l_7^m$,

ψ= $\{(e_1, e_3), \{(\times_1, 0.1), (\times_2, 0), (\times_3, 0.3), (\times_4, 0)\}), ((e_1, e_4), \{(\times_1, 0.1), (\times_2, 0), (\times_3, 0), (\times_4, 0.4)\}),$

$((e_2, e_3), \{(\times_1, 0), (\times_2, 0.2), (\times_3, 0.3), (\times_4, 0)\}, ((e_2, e_4), \{(\times_1, 0), (\times_2, 0.2), ((\times_3, 0), (\times_4, 0.4)\}$, to find positive, negative and boundary BSF binary soft fuzzy set.

Pos$\psi(N)$ =$\underline{\psi}$ (N)

Neg $\psi(N) = \mu - \bar{\psi}$ (N) =$(\psi(N))^c$;

Bound $\psi(N) = \bar{\psi}$ (N)- $\underline{\psi}$ (N).

Pos$\psi(N)$=$\{ (e_1, e_3), \{(\times_1, 0.1), (\times_2, 0), (\times_3, 0.3)\}, ((e_1, e_4), \{(\times_2, 0), (\times_3, 0)\},$

$((e_2, e_3), \{(\times_1, 0), (\times_2, 0.2), (\times_3, 0.3)\}, ((e_2, e_4), \{(\times_2, 0), (\times_3, 0)\},$

Neg $\psi(N)$ =$\{(e_1, e_3), \{(\times_2, 0), (\times_4, 0)\}), ((e_1, e_4), \{(\times_1, 0.1)\}),$

$((e_2, e_3), \{(\times_1, 0), (\times_4, 0)\}, ((e_2, e_4), \{(\times_1, 0), (\times_3, 0), (\times_4, 0.4)\}$

Bound $\psi(N)$= $\{(e_1, e_3), \{0\}), ((e_1, e_4), \{ 0\}), ((e_2, e_3), \{0\}, ((e_2, e_4), \{0\}\}$ is BSF definable.

$$M_{BSF}(N) = M_\psi(N) = \left|\frac{\underline{\psi}\,(N)}{\bar{\psi}\,(N)}\right| = \left|\frac{((e_1,e_3),\{(\times_1,0.1),((\times_2,0),((\times_3,0.3)\})((e_1,e_4),\{((\times_1,0.1),(\times_2,0),((\times_3,0)\}), \; ((e_2,e_3),\{(\times_1,0),(\times_2,0.2),((\times_3,0.3)\},,(e_2,e_4),\{(\times_1,0),((\times_2,0.2),((\times_3,0)\})}{((e_1,e_3),\{(\times_1,0.1),((\times_3,0.3)\})((e_1,e_4),\{((\times_1,0.1)\}), \; ((e_2,e_3),\{(\times_2,0.2),((\times_3,0.3)\},,(e_2,e_4),\{(\times_2,0.2)\})}\right| = \left| ((e_1, e_3), \{(\times_2, 0)\}), ((e_1, e_4), \{((\times_2, 0), ((\times_3, 0)\}, (e_2, e_3), \{(\times_1, 0)\})), (e_2, e_4), \{(\times_1, 0), ((\times_3, 0)\} \right|.$$

is BSF accuracy. To represented example above in MATLAB we get three figures, this three method different to represented data's by method accuracy in MATLAB in **Figure 1 (a, b, c)**.

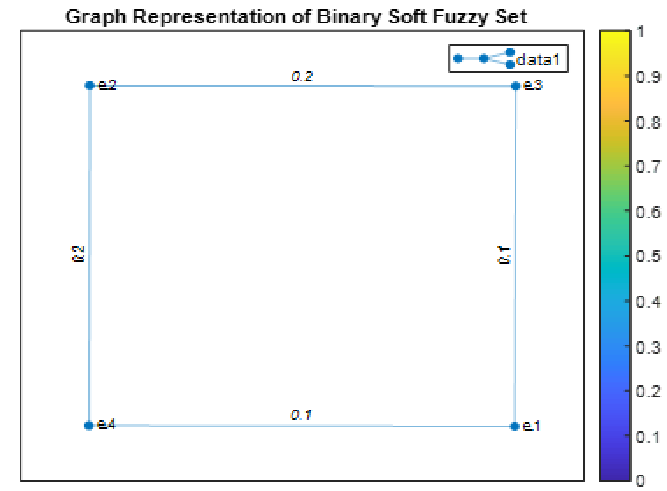

Figure 1. a: Graph Representation of Binary Soft Fuzzy Set.

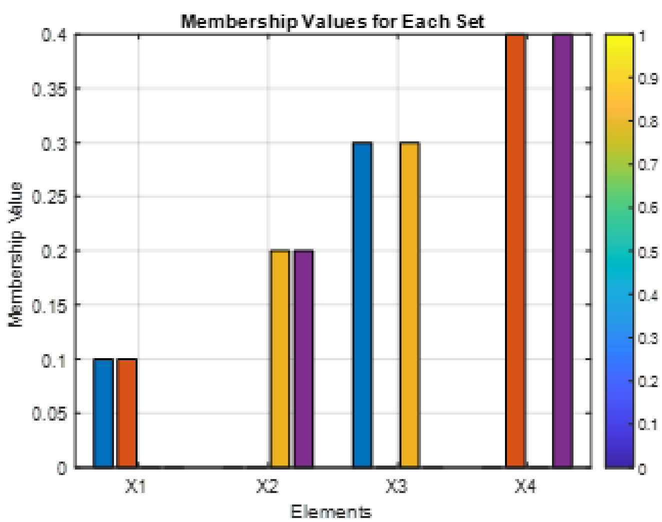

Figure 1. b: Membership Values for Each Set.

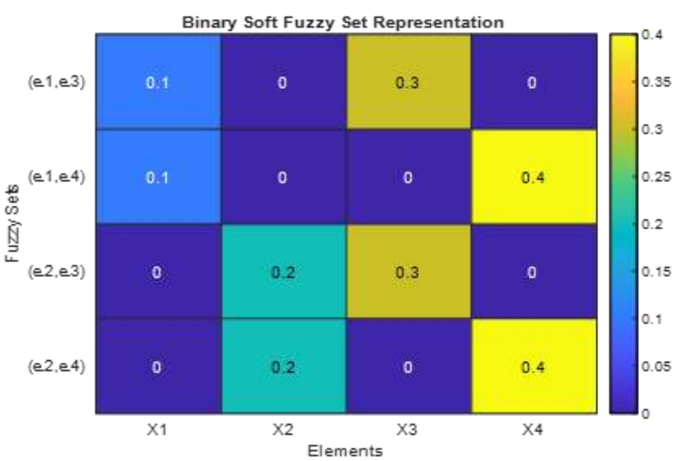

Figure 1. c: Binary Soft Fuzzy Set non-accuracy Representation.

3. 5 Proposition:

Let $(\psi, £_1 \times £_2)$ be BSF over μ and a pair (μ, ψ) is BSF approximation space for $\forall \times_i, \times_j \tilde{\sqsubseteq} \mu$, then following are holds:

1-$\underline{\psi}\,(\bar{\bar{0}})= \bar{\psi}\,(\bar{\bar{0}}\,) =\bar{\bar{0}}$,

2-$\underline{\psi}\,(\bar{\bar{1}})= \bar{\psi}\,(\bar{\bar{1}}\,) =\bar{\bar{1}}$,

3-$\bar{\psi}\,(\mu)=\underline{\psi}(\mu) = \sqcup^{(\psi)}_{(e_i,e_j)\in £_1\times £_2}$,

4-$\underline{\psi}(\times_i \sqcap \times_j) \tilde{\sqsubseteq} \underline{\psi}\left(\times_i\right)\sqcap\underline{\psi}(\times_j)$,

5-$\underline{\psi}\,(\times_i)\sqcup\underline{\psi}(\times_j) \tilde{\sqsubseteq} \underline{\psi}\,(\times_i \sqcup \times_j)$,

6-$\bar{\psi}(\times_i \sqcup \times_j) = \bar{\psi}(\times_i)\sqcup\bar{\psi}(\times_j)$,

7-$\bar{\psi}\;(\times_i \sqcap \times_j) \tilde{\sqsubseteq} \bar{\psi}(\times_i)\sqcap\bar{\psi}(\times_j)$,

8- if $\times_i \tilde{\sqsubseteq} \times_j$ then $\underline{\psi}(\times_i) \tilde{\sqsubseteq} \underline{\psi}(\times_j)$,

9- if $\times_i \tilde{\sqsubseteq} \times_j$ then $\bar{\psi}\;(\times_i) \tilde{\sqsubseteq} \bar{\psi}\;(\times_j)$,

Proof: to prove 1,2

since BS lower $(\emptyset) = BS\ upper(\emptyset) = \emptyset$ ……….1

and Bf lower $(\tilde{0}) = Bf\ upper(\tilde{0}) = \tilde{0}$ ………… 2

and from1and 2 we get $\underline{\psi}\,(\bar{\bar{0}})= \bar{\psi}\,(\bar{\bar{0}}\,) =\bar{\bar{0}}$,

By the same way we get $\underline{\psi}\,(\bar{\bar{1}})= \bar{\psi}\,(\bar{\bar{1}}\,) =\bar{\bar{1}}$.

3-To prove $\bar{\psi}\,(\mu)=\underline{\psi}(\mu) = \sqcup^{(\psi)}_{(e_i,e_j)\in £_1\times £_2}$,

$\bar{\psi}\,(\mu)= \mu\sqcap\sqcup^{(\psi)}_{(e_i,e_j)\in £_1\times £_2} = \sqcup^{(\psi)}_{(e_i,e_j)\in £_1\times £_2}$,

4-Since $(\times_i \sqcap \times_j) \tilde{\sqsubseteq} \times_i$ and $(\times_i \sqcap \times_j) \tilde{\sqsubseteq} \times_j$,

$\underline{\psi}(\times_i \sqcap \times_j) \tilde{\sqsubseteq} \underline{\psi}(\times_i)$ and $\underline{\psi}(\times_i \sqcap \times_j) \tilde{\sqsubseteq} \underline{\psi}(\times_j)$, then by 8 $\underline{\psi}(\times_i \sqcap \times_j) \tilde{\sqsubseteq} \underline{\psi}\left(\times_i\right)\sqcap\underline{\psi}(\times_j)$.

5- To prove $\underline{\psi}\,(\times_i)\sqcup\underline{\psi}(\times_j) \tilde{\sqsubseteq} \underline{\psi}\,(\times_i \sqcup \times_j)$,

Since $\times_i \tilde{\sqsubseteq} \times_i \sqcup \times_j$, and $\times_j \tilde{\sqsubseteq} \times_i \sqcup \times_j$,

by 9 $\underline{\psi}(\times_i) \tilde{\sqsubseteq} \underline{\psi}(\times_i \sqcup \times_j), \underline{\psi}(\times_j) \tilde{\sqsubseteq} \underline{\psi}(\times_i \sqcup \times_j)$,

$\underline{\psi}\,(\times_i)\sqcup\underline{\psi}(\times_j) \tilde{\sqsubseteq} \underline{\psi}\,(\times_i \sqcup \times_j)$.

6-To prove $\bar{\psi}(\times_i \sqcup \times_j) = \bar{\psi}(\times_i)\sqcup\bar{\psi}(\times_j)$,

since $\times_i \tilde{\sqsubseteq} \times_i \sqcup \times_j$, and $\times_j \tilde{\sqsubseteq} \times_i \sqcup \times_j$,

by 9, $\bar{\psi}(\times_i) \tilde{\sqsubseteq} \bar{\psi}(\times_i \sqcup \times_j), \bar{\psi}(\times_j) \tilde{\sqsubseteq} \bar{\psi}(\times_i \sqcup \times_j)$ ………...1

From definition $\bar{\psi}$ (N)=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \cap N \neq \bar{\bar{0}}$]},

$\bar{\psi}$ ($\times_i\ \sqcup \times_j$)=$\sqcup_{(}e_i$, e$_j$){ $\times_i\ \sqcup \times_j \in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times_i \sqcup \times_j \in (\psi_i \sqcup \psi_j\ ) \cap N \neq \bar{\bar{0}}$]},

= $\psi_i\ \cap N \neq \bar{\bar{0}} \vee\ \psi_j\ \cap N \neq \bar{\bar{0}}$

$\widetilde{\sqsubseteq}\ \bar{\psi}(\ \times_i) \sqcup \bar{\psi}(\times_j)$ … … … … … … .2, from 1,2 we get

$\bar{\psi}(\times_i \sqcup \times_j) = \bar{\psi}(\times_i) \sqcup \bar{\psi}(\times_j)$.

7- Since $(\ \times_i \sqcap \times_j)\ \widetilde{\sqsubseteq} \times_i$ and $(\ \times_i \sqcap \times_j)\ \widetilde{\sqsubseteq} \times_j$,

since by 8, $\bar{\psi}(\ \times_i \sqcap \times_j)\ \widetilde{\sqsubseteq}\ \bar{\psi}(\times_i)$ and $\bar{\psi}(\ \times_i \sqcap \times_j)\ \widetilde{\sqsubseteq}\ \bar{\psi}(\ \times_j)$,

$\bar{\psi}(\ \times_i \sqcap \times_j)\ \widetilde{\sqsubseteq}\ \bar{\psi}(\ \times_i) \sqcap \bar{\psi}(\times_j)$.

8- If $\times_i \widetilde{\sqsubseteq} \times_j$, $\underline{\psi}(\ \times_i)$=$\sqcup_{(}e_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times_i \in f_{\psi i}\ \widetilde{\sqsubseteq}\ N$]},

$\underline{\psi}(\ \times_j)$=$\sqcup_{(}e_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times_j \in \psi_j\ \widetilde{\sqsubseteq}\ N$]},

since $\times_i \widetilde{\sqsubseteq} \times_j$, so $\underline{\psi}(\ \times_i)\ \widetilde{\sqsubseteq}\ \underline{\psi}(\times_j)$.

9- If $\times_i \widetilde{\sqsubseteq} \times_j$, $\bar{\psi}(\ \times_i)$=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times_i \in (\psi_i\ ) \cap N \neq \bar{\bar{0}}$]},

$\bar{\psi}(\ \times_j)$=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times_j \in (\psi_j\ ) \cap N \neq \bar{\bar{0}}$]},

since $\times_i \widetilde{\sqsubseteq} \times_j$, so $\bar{\psi}(\ \times_i)\ \widetilde{\sqsubseteq}\ \bar{\psi}(\times_j)$.

3. 6 Proposition:

Let $(\psi, £_1 \times £_2)$ be BSF over μ , the pair (μ, ψ) is BSF approximation space for the for each $N\ \widetilde{\sqsubseteq}\ \mu$, then following are holds:

1-$\underline{\psi}$ ($\underline{\psi}(N)$)=$\underline{\psi}$ *(N)* ,

2-$\bar{\psi}$ $(N)\ \widetilde{\sqsubseteq}\ \bar{\psi}$ ($\bar{\psi}(N)$),

3-$\underline{\psi}$ $(N)\ \widetilde{\sqsubseteq}\ \bar{\psi}$ ($\underline{\psi}(N)$),

4-$\underline{\psi}(\ \bar{\psi}(\ (N))\ \widetilde{\sqsubseteq}\ \bar{\psi}(N))$.

Proof:

1-To prove $\underline{\psi}$ ($\underline{\psi}(N)$)=$\underline{\psi}$ *(N)* from definition BSF lower approximation,

$\underline{\psi}$ (N)=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \widetilde{\sqsubseteq}\ N$]},

by using BSF lower of $\underline{\psi}$ *(N)* anther, hence $\underline{\psi}$ ($\underline{\psi}(N)$)=$\underline{\psi}$ *(N)*.

This is clear equality in example 3.2.

2- To prove $\bar{\psi}$ $(N)\ \widetilde{\sqsubseteq}\ \bar{\psi}$ ($\bar{\psi}(N)$), The definition of BSF upper approximation,

$\bar{\psi}$ *(N)*=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \cap N \neq \bar{\bar{0}}$]},

by using BSF upper of $\bar{\psi}$ *(N)* anther, since $\psi\ \cap N \neq \bar{\bar{0}}\ \widetilde{\sqsubseteq}$ ($\psi\ \cap N(\psi\ \cap N \neq \bar{\bar{0}})$) , hence $\bar{\psi}$ $(N)\ \widetilde{\sqsubseteq}\ \bar{\psi}$ ($\bar{\psi}(N)$).

3- To prove $\underline{\psi}$ $(N)\ \widetilde{\sqsubseteq}\ \bar{\psi}$ ($\underline{\psi}$ (N)), from definition BSF lower approximation, $\underline{\psi}$ (N)=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \widetilde{\sqsubseteq}\ N$]},using BSF upper $\underline{\psi}$ (N)

$\bar{\psi}$ *(N)*=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \cap N \neq \bar{\bar{0}}$]}, since$\times \in \psi\ \widetilde{\sqsubseteq}\ N$]} $\widetilde{\sqsubseteq}$ ($\psi\ \cap N(\psi\ \cap N \neq \bar{\bar{0}})$), hence $\underline{\psi}$ $(N)\ \widetilde{\sqsubseteq}\ \bar{\psi}$ ($\underline{\psi}$ (N))

4- To prove $\underline{\psi}(\bar{\psi}$ $(N))\ \widetilde{\sqsubseteq}\ \bar{\psi}$ (N) from definition BSF lower approximation, $\underline{\psi}$ (N)=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \widetilde{\sqsubseteq}\ N$]},using BSF upper $\underline{\psi}$ (N)

$\bar{\psi}$ *(N)*=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \cap N \neq \bar{\bar{0}}$]}, since$\times \in \psi\ \widetilde{\sqsubseteq}\ N$]} ($(\psi\ \cap N \neq \bar{\bar{0}})$) $\widetilde{\sqsubseteq} \times \in \psi\ \cap N \neq \bar{\bar{0}}$, hence $\underline{\psi}(\bar{\psi}$ $(N))\ \widetilde{\sqsubseteq}\ \bar{\psi}$ (N)

3. 7 Proposition:

Let $(\psi, £_1 \times £_2)$ be BSF over μ ,The pair (μ, ψ) is BSF approximation space for the for each $N\ \widetilde{\sqsubseteq}\ \mu$, then following are holds:

1-$\underline{\psi}$ (μ))=μ ,

2-$\underline{\psi}(\ \bar{\psi}(\ (N)) = \bar{\psi}(N)\ , \forall N\ \widetilde{\sqsubseteq}\ \mu$.

Proof:

1- Let $(\psi, £_1 \times £_2)$ be BSF over μ, then by proposition 2.4 . By (1) to prove $\underline{\psi}$ (μ))=μ

$\underline{\psi}$ (μ)=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \widetilde{\sqsubseteq}\ \mu$]} $=\ \mu$

2- By proposition 3.5 we get $\underline{\psi}(\ \bar{\psi}(\ (N))\ \subseteq \bar{\psi}(N)\ , \forall N\ \widetilde{\sqsubseteq}\ \mu$ … … … … … . .1. The second direction, since $N\ \widetilde{\sqsubseteq}\ \bar{\psi}(N)\ , \forall N\ \widetilde{\sqsubseteq}\ \mu$. By proposition 2.5 and proposition 2.4, hence $N\ \widetilde{\sqsubseteq}\ \underline{\psi}(\ \bar{\psi}($ $(N)\ , \forall N\ \widetilde{\sqsubseteq}\ \mu$. So that $\underline{\psi}(\ \bar{\psi}(\ (N))\ \widetilde{\sqsubseteq}\ \bar{\psi}(N))$ and

$\bar{\psi}$ *(N)*=$\sqcup_{(}$e$_i$, e$_j$){ $\times\in\ \mu, \exists$ e$_i$, e$_j\ \in £_1 \times £_2$, [$\times \in \psi\ \cap N \neq \bar{\bar{0}}$] $\widetilde{\sqsubseteq}\ \underline{\psi}(\ \bar{\psi}(\ (N)$,

So that $\bar{\psi}(N)\ \subseteq\ \underline{\psi}(\ \bar{\psi}(\ (N))$……………..2, from 1,2 then $\underline{\psi}($ $\bar{\psi}(\ (N)) = \bar{\psi}(N)\ , \forall N\ \widetilde{\sqsubseteq}\ \mu$.

This proposition 3.4 we represent BSF lower approximation and BSF upper approximation in Table 1 from feng approach.

This answers the value between $\bar{\bar{0}}, \bar{\bar{1}}$, table 1:

Table 1: BSF lower approximation and BSF upper approximation.

Nu.	$\underline{\psi}$(N)	$\bar{\psi}(N)$
1	$\bar{\bar{1}}$	$\bar{\bar{1}}$
2	$\bar{\bar{1}}$	$\bar{\bar{1}}$
3	$\bar{\bar{0}}$	$\bar{\bar{0}}$
4	$\bar{\bar{0}}$	$\bar{\bar{1}}$
5	$\bar{\bar{1}}$	$\bar{\bar{1}}$
6	$\bar{\bar{1}}$	$\bar{\bar{1}}$
7	$\bar{\bar{1}}$	$\bar{\bar{1}}$
8	$\bar{\bar{1}}$	$\bar{\bar{1}}$
9	$\bar{\bar{1}}$	$\bar{\bar{1}}$

4 Future work

we work – on fuzzy (minimal, connected), binary soft fuzzy of minimal approximation and binary soft fuzzy of connected approximation.

5 Conclusion

The Soft Fuzzy set is a new field of development and with its help not only new decision-making models and problems via the interactive soft computing complexity and time analysis but new mathematical models may be evolved which will be incentive to the natural sciences such as information systems, biomathematics and other. The novel structure of binary soft fuzzy approximation space is introduced in this paper. An attempt has been made to discuss some of the basic notions and concepts. the purpose of this paper is just to introduce the theorem and there are many opportunities for investigation for the researchers in this area, i. e. this is starting of some new kind of soft fuzzy approximation space structure and the such concepts like binary soft fuzzy lower and BSF upper and another basic concepts as, separation principles, and other things can be diverse about this new type of soft fuzzy reliability.

References

[1] Al-Swidi, L. A. A., & Saeed, A. A. Soft fuzzy and fuzzy soft fuzzy theory with applications. *Department of Mathematics, College of Education for Pure Science, University of Babylon* (2020).

[2] Cetkin, V., & Aygun, H. A note on fuzzy topological spaces. *Proceedings of the 8th Conference of European Society for Fuzzy Logic and Technology (EUSFLAT)*, 56–60 (2013).

[3] Chang, C. L. Fuzzy topological spaces. *Journal of Mathematical Analysis and Applications* **24**, 182–190 (1968).

[4] E. Rosas, C. Carpintero, J. Sanabria. Characterization of I-b-open sets, *Journal of Interdisciplinary Mathematics* ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online) **26**(2), 261–269 doi: 10.47974/JIM-1466 (2023).

[5] Feng, F., Liu, X., Fotea, V. L., & Jun, Y. B. Soft sets and soft rough sets. *Information Sciences*, **181**(6), 1125–1137. doi: 10.1016/j.ins.2010.11.004 (2011).

[6] Halgwrd M. Darwesh1, Sarhad F. Namiq2. Pre-irresolute functions in closure spaces, *EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS* **12**(3), 1082-1095 ISSN 1307-5543 (2019).

[7] Kalpana, G., & Kalaivani, C. Fuzzy soft topology. *International Journal of Engineering Studies* **9**(1), 45–56 (2017).

[8] Maji, P. K., Biswas, R., & Roy, A. R. Fuzzy soft sets. *Journal of Fuzzy Mathematics*, **9**(3), 589–602 (2011).

[9] Ming, P. P., & Ming, L. Y. Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith convergence. *Journal of Mathematical Analysis and Applications* **76**, 571–599 (1980).

[10] Molodtsov, D. Soft Set Theory – First results. *Computers and Mathematics with Applications* **37**(4/5), 19–31 (1999).

[11] Mostafa, K. E.-B., & El Atik, A. F. A. Soft B-rough sets and their application to determine COVID-19. *Turkish Journal of Mathematics* **45**, 1133–1148. doi: 10.3906/mat-2008-93 (2021).

[12] P. Gino Metilda and J. Subhashini, Remarks on fuzzy binary soft limit points, *AIP Conference Proceedings* **2385**, 130035, doi: 10.1063/5.0070723 (2022).

[13] Pawlak, Z. Rough sets. *International Journal of Computing and Information Sciences* **11**(5), 311–356, doi: 10.1007/BF01001956 (1982).

[14] Roy, S., & Samanta, T. K. A note on fuzzy soft topological spaces. *Annals of Fuzzy Mathematics and Informatics* **3**(2), 305–311 (2011).

[15] Saber, Y. M., & Abdel-Sattar, M. A. Ideal on fuzzy topological spaces. *Applied Mathematical Sciences* **8**(34), 1667–1691 (2014).

[16] Sangoor, A. A., & Hassan, A. F. Binary Soft Fuzzy Sets. *AIP Conference Proceedings* (Accepted for publication) (2024).

[17] Sarhad F. Namiq, Ennis Rosas. On $\lambda^D - R_0$ and $\lambda^D - R_1$ spaces. *Acta Univ. Sapientiae, Mathematica* **12**(2), 307–316 doi: 10.2478 (2020).

[18] Sarhad F. Namiq, Ennis Rosas. New types of connected, component spaces and maximal open set via s-operation, *Journal of Interdisciplinary Mathematics*, ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online) **27**(3), 481–495 doi: 10.47974/JIM-1723 (2024).

[19] Serkan, A., & Idris, Z. On fuzzy soft topological spaces. *Annals of Fuzzy Mathematics and Informatics* **5**(2), 377–386 (2013).

[20] Shabir, M., & Naz, M. On soft topological spaces. *Computers and Mathematics with Applications* **61**(7), 1786–1799 (2011).

[21] Sostak, A. P. On fuzzy topological structure. *Supplemento ai Rendiconti del Circolo Matematico di Palermo*, Serie II, **11**, 89–103 (1985).

[22] Tany, B., & Kandemir, M. B. Topological structures of fuzzy soft sets. *Computers and Mathematics with Applications* **61**, 412–418 (2011).

[23] Varol, B. P., & Aygun, H. Fuzzy soft topology. *Hacettepe Journal of Mathematics and Statistics* **41**(3), 407–419 (2012).

[24] Zadeh, L. A. Fuzzy sets. *Information and Control* **8**, 338–353 (1965).