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Original Article

VARMA Time Series Model Analysis Using DWT Coefficients for Coiflets Wavelet

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ABSTRACT

Multivariate time series models (Vector Autoregressive Moving Average models) are complex models and some traditional methods, such as optimization functions, are used to estimate the model parameters, but they are far from their true values. Therefore, approximate VAR models are usually used to get results closer to the true values of the autoregressive parameters. This article proposes to improve the performance of approximate VAR models through wavelet analysis, specifically Coiflets wavelet. The proposed method is based on the discrete wavelet transform (DWT) (Coiflets wavelet) which divides the time series data into two parts the first part represents the approximation coefficients (low-pass filter), while the second part represents the detail coefficients (high-pass filter). Maximum likelihood estimation is used to estimate the parameters of the two approximate VAR models through which approximation and detail parameters are predicted. The inverse discrete wavelet transforms of the estimated coefficients of the Coiflets wavelet at orders (1, 2, ...,5) are used to obtain filtered data, which will be used to estimate the parameters of the approximate VAR models. The efficiency and accuracy of the estimated parameters using the classical and proposed methods were compared through simulation and real data based on monthly Iraqi economic data from January 2010 to July 2024, covering 175 observations and focusing on key indicators such as Official Reserves (OR) and Broad Money (M2). Evaluation metrics included mean square error (MSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) . The analysis, conducted using tools like using Minitab V21, EViews V12, MATLAB R2023a, and custom MATLAB programs. The approximate VAR models of the proposed method were more efficient than the classical method.

Keywords: VARMA Model, Approximate VAR Model, Discrete Wavelet Transformation, Coiflets Wavelet, and Low and High Pass Filters.

1 Introduction

Vector Autoregressive Moving Average (VARMA) modeling for multiple time series is an advanced statistical technique that is used to examine the temporal dynamics between multiple variables and their effects over time. These models combine features of Autoregressive (AR) and Moving Average (MA) models to achieve efficient and robust estimates of time-dependent variables. However, VARMA models are computationally complex and difficult to implement, especially in estimating optimal parameter values. Typical methods use optimization algorithms to estimate parameters, but these methods can yield unreliable estimates because they are more sensitive to random noise in the data [1].

Since the parameters of VARMA models are difficult to estimate, approximate VAR models are preferred as a more efficient alternative for more precise estimation. These models are especially beneficial for noisy time series data, where noise degrades the quality of estimates. The approximate VAR model is a statistically powerful technique preferred to reduce computational errors through advanced optimization methods that minimize the difference between actual values and their predictions [2,3]. Wavelet analysis, as one example, has become a contemporary technique used to achieve more accurate approximations of VAR models, since it enables efficient modeling of noise and fine details within time series data.

Recent studies have highlighted the growing interest in VARMA models due to their flexibility and robustness in capturing complex temporal dependencies in multivariate time series data. These models have been shown to effectively model shifting associations between past observations and random shocks, making them a valuable tool for multivariate analysis[4,5].

Moreover, recent research has demonstrated the flexibility and robustness of VARMA models to capture intricate temporal structures, enable Granger to cause analysis and more accurate

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forecasting, and enhance structural modeling [4]. For instance, investigated the links between Dynamic Factor Models and VARMA representations to provide new insight on the problem of dimensionality reduction and solutions of nonlinear matrix equations [6]. In the field of economics, VARMA models are extensively used to predict macroeconomic indicators and to describe financial yield curves [7]. A modified information criterion has been constructed to select the optimal order of a VARMA model, which is based on prior work by Hannan & Rissanen (1982) and Hannan & Kavalieris (1984b). This improved technique boosts the efficiency of estimations and leads to a more reliable model selection process [8].

Recent techniques have been developed to promote the use of VARMA models in high dimensions. For example, Huang et al. (2024) offered the Scalable ARMA (SARMA) model that applies tensor decomposition techniques to jointly estimate VARMA processes among massive time series data[9]. In addition, Li & Yang (2024) used a dual modeling technique called DeepVARMA which integrates a neural network with VARMA for better predictive accuracy on the chemical industry index [5].

Thus innovations highlight the extensive applicability of VARMA as an important analytical tool in many fields, such as economics, finance, and engineering, and its usefulness in improving the accuracy of estimates and facilitating the analysis of multivariate time series data.

Wavelet analysis has demonstrated its usefulness in modeling time series by breaking down data into components based on time and frequency, which helps in minimizing noise and identifying trends. thereby aiding in the reduction of noise and the identification of trends. Studies conducted by [10] along with [11] have shown that utilizing wavelet methods enhances the accuracy of parameter estimation in economic and financial time series by distinguishing between short-term and long-term patterns.

The research problem is that the estimation of VARMA model parameters is computationally difficult, and sample time series are noisy and subject to random variability, causing problems in estimating these models. Nonlinear elements and noise frustrate the efficacy of traditional optimization methods to achieve reliable estimates.

To address this issue this study employs the Discrete Wavelet Transform (DWT) to the time series before estimating approximate VAR models to improve accuracy and efficiency in estimation by reducing noise and separating general trends from fluctuations.

The study also evaluates the proposed method against conventional methods using Mean Squared Error (MSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC).

The evaluation tests the improvement of wavelet-based methods in estimating the VARMA model for economic and financial data analysis.

Wavelet analysis provides a powerful framework for signal and time series analysis broadly, allowing for the decomposition of data into multiple levels of frequency to improve estimation. Coiflets, a specific family of wavelets, are highly effective because they optimize for accuracy in both time and frequency, which makes them desirable for complex analysis of time series data [10].

The proposed method leverages the Discrete Wavelet Transform (DWT), which separates time series data into two general components:

- Approximation coefficients flow through a low-pass filter and capture general trends.
- Detail coefficients, filtered through a high-pass filter, capture short-term oscillations and fluctuations.

This decomposition is useful for filtering out noise and isolating dominant data patterns that can improve parameter estimation in approximate VAR models[11].

The approximate VAR models are separately estimated for approximation and detail coefficients using maximum likelihood estimation. The filtered data is then reconstructed using the inverse Discrete wavelet transform for further, now more effective, analyses of the time series. Simulations and real data were used to compare the efficiency and accuracy of estimated parameters between the traditional and proposed methods.

Wavelet augmented models outperformed conventional models, confirming the efficacy of the wavelet method in extracting better estimates for the VARMA models. Further studies confirmed that the use of Discrete Wavelet Transform in concert with approximate VAR models improved statistical precision by minimizing the effects of noise, offering a powerful tool of analysis for a broad range of fields such as economics, finance, and [10].

The research focused on two main components including the theoretical component, which dealt with the VARMA model, the approximate VAR Model, Model Selection Criteria, and the proposed method. While the applied components, also included a case study on simulation and real data from the Iraqi economy, to verify the algorithm's validity. Statistical tools were used, including Minitab V21, EViews V12, and MATLAB R2023a, along with specially designed programs in MATLAB program. The study concludes with results drawn from both simulations and practical real-world applications.

2 Theoretical Aspect

The theoretical component introduced fundamental ideas regarding research from a statistical perspective, as outlined in the following sections.

2.1 VARMA Model

The Vector Autoregressive Moving Average (VARMA) model is a sophisticated tool for examining the dynamic relationships of multiple time series. The model incorporates the properties of both AR and MA processes to form a broader specification of temporal dependencies and multivariate interrelationships. Given the focus of this study on enhancing approximate VAR models, the broader VARMA structure is adopted as a general framework, which allows flexibility in modeling both autoregressive and moving average dynamics [2], enabling more accurate decomposition when combined with wavelet analysis.

Suppose y_t is a stationary k-dimensional time series with a mean of zero ($\mu_{y_t} = 0$). The mathematical representation of the VARMA(p, q) model is given by :

$$y_t = \sum_{i=1}^{p} \Phi_i \, y_{t-i} + \sum_{m=1}^{q} \Theta_m \, e_{t-m} + e_t \tag{1}$$

Where y_t is a $k \times 1$ vector of observed time series values at time t, Φ_i are $k \times k$ Autoregressive (AR) parameter matrices of order p, Θ_m are $k \times k$ Moving Average (MA) parameter matrices of order q, e_t is a $k \times 1$ vector of error terms (innovations), which follows a white noise process with a covariance matrix (Σ_e) of dimension $k \times k$.

Alternatively , it can be written using the Backshift Opera B, as follows :

$$\Phi(B) y_t = \Theta(B) e_t \tag{2}$$

Where $\Phi(B) = I - \sum_{i=1}^{p} \Phi_i \, B^i$ represents the autoregressive polynomial matrix, $\Theta(B) = I + \sum_{m=1}^{q} \Theta_m \, B^m$ represents the moving average polynomial matrix, B represents the Backshift Operator B, where $B^k y_t = y_{t-k}$, I is a $k \times k$ identity matrix, so that the model has the correct dimensions ande_t is the vector of errors (innovations) at time t.

The model efficiently models the shifting associations between past time series data and random shocks, making the VARMA model a powerful choice for multivariate time series analysis [5,11].

Recent research has demonstrated the flexibility and robustness of VARMA models to capture intricate temporal structures, enable Granger to cause analysis and more accurate forecasting, and enhance structural modeling [11]. Bhamidi et al. (2023) investigated the links between Dynamic Factor Models and VARMA representations to provide new insight on the problem of dimensionality reduction and solutions of nonlinear matrix equations [6] . In the field of economics, VARMA models are extensively used to predict macroeconomic indicators and to describe financial yield curves [7]. A modified information criterion has been constructed to select the optimal order of a VARMA model, which is based on prior work by Hannan & Rissanen (1982) and Hannan & Kavalieris (1984b). This improved technique boosts the efficiency of estimations and leads to a more reliable model selection process [8].

Recent techniques have been developed to promote the use of VARMA models in high dimensions. For example, Huang et al. (2024) offered the Scalable ARMA (SARMA) model that applies tensor decomposition techniques to jointly estimate VARMA processes among massive time series data [13] . In addition, Li & Yang (2024) used a dual modeling technique called DeepVARMA which integrates a neural network with VARMA for better predictive accuracy on the chemical industry index [5].

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2.2 Approximate VAR Model

The Approximate Vector Autoregressive (Approximate VAR) model is an extension of the Vector Autoregressive Moving Average (VARMA) model and was developed to facilitate estimation by replacing the Moving Average (MA) component with an extended Autoregressive (AR) component. The result is a model that is less computationally intensive than when the MA components are present but that can still accurately evaluate the behavior of economic and financial time series data. As a result, users can conduct more accurate and stable dynamic analyses, especially in cases where the estimation of MA parameters would be difficult or unnecessary [12].

Based on equation (1) of the mathematical expression for the VARMA(p,q) model, the approximate VAR model eliminates the Moving Average (MA) terms and raises the number of autoregressive (AR) lags to take their place, leading to the following equation as given [8,13] .

According to the mathematical representation for the VARMA(p, q) model shown in equation (1), the Approximate VAR model removes the Moving Average (MA) components and increases the Autoregressive (AR) order lag to compensate for their absence, as given in the following equation [8,13].

$$y_t = \sum_{i=1}^{p^*} \Phi_i^* \, y_{t-i} + e_t^* \tag{3}$$

Where y_t is a $k \times 1$ vector that represents the values of the observed time series at time t, $p^* > p$ is an extended autoregressive (AR) order to replace the MA components, Φ_i^* are the new autoregressive parameters after expansion, e_t^* is the new error term that is assumed to be uncorrelated over time t.

The Approximate VAR Model is one of the most common methodology used for economic and financial research because of its efficiency and analytical strength. From thought removing the (MA) component, this model decreases the number of parameters needed for estimation, which streamlines calculations and improves efficiency [14]. It also means that standard estimation methods (such as Ordinary Least Squares) are generally more practical than with alternative models, especially when dealing with large economic datasets [8]. Moreover, increasing the order of the Autoregressive (AR) part increases the model's predictive ability, which enables the model to better understand structural shocks and longer-term economic trends [14]. These characteristics make the Approximate VAR Model a reliable and powerful model for analyzing time series data, particularly in complex data analysis scenarios where computational efficiency is important .One common use of the model is in macroeconomic research, financial market prediction, and economic policy analysis, helping researchers and policymakers analyze the effects of fiscal policy, market dynamics, and the business cycle [13].

Overall , the Approximate VAR Model offers a computationally efficient and analytically robust method for analyzing multivariate time series data. By balancing the trade-off between ease of estimation and forecasting performance, the model remains the go-to choice for academic researchers and industry professionals in economics and finance.

2.3 Model Selection Criteria

Employ AIC and BIC criteria as essential factors in model selection by balancing model accuracy and complexity to determine the best model from a set of alternatives, as defined below [15,16] :

Akaike Information Criterion (AIC)

A lower AIC value signifies a superior model, indicating an optimal balance between goodness of fit and model simplicity. This is computed using the following formula :

$$AIC = 2k - 2Ln(L) \tag{4}$$

Where k is the number of parameters in the model, with (k = 1, 2, and 3), L denotes the log-likelihood function of the model.

A lower AIC value suggests a more efficient model, as it reflects a better fit with fewer parameters.

Schwarz Information Criterion (SIC or BIC)

The Bayesian Information Criterion, often referred to as the Schwarz information criterion (also known as SIC, SBC, and SBIC), serves as an alternative criterion for choosing models. It evaluates both the fit's adequacy and the model complexity but imposes a stricter penalty related to the number of parameters in comparison to the AIC. This is computed using the following formula:

$$BIC = kLn(n) - 2Ln(L) \tag{5}$$

Where n is the number of observations, k is the number of parameters in the model, with (k = 1, 2, and 3), L is the log-likelihood function of the model.

Similar to AIC, a lower BIC value points to the best model however, BIC generally prefers simpler models over AIC because of the $Ln(n)$ term.

Mean Square Error (MES)

The Mean Square Error (MSE) is a statistical measure used to determine the predictive power of a model, as it is the most commonly used metric for assessing model performance and selecting the most efficient predictive model with the lowest error for future predictions. It is widely applied in regression analysis, machine learning, and time series forecasting.

Mathematically, MSE is defined as follows [17,18]:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(e_t)^2 = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2 \tag{6}$$

Where y_t represents the actual value of the time series at time t, $\hat{y}_t$ represents the predicted value of the time series at time t, $e_t = y_t - \hat{y}_t$ represents the prediction error at time t., n represents the sample size or the number of actual observations.

Summary, a lower **MSE** indicates a more accurate model, while a higher value suggests greater prediction errors which implying inefficiencies in the model.

3 Discrete Wavelet Transform (DWT)

The Discrete Wavelet Transform (DWT) is a powerful mathematical tool for signal analysis, enabling representation in both time and frequency domains. DWT is widely used in digital signal processing, image compression, and feature extraction for pattern recognition [19] .

In the discrete wavelet transform (DWT) process, the original series or data is divided into two components [22] :

1. Mother Wavelet- representing the low-frequency components and also referred to as the approximation part or the approximation coefficients (smoothing), because it

gives the data in its uniform form, highlighting the general trends of the signal.

2. Father Wavelet- representing the high-frequency components and often representing noise or unwanted variations in the data, commonly referred to as Detail coefficients, as it reveals fine variations and sharp transitions within the signal.

Mathematically, the core of the Discrete Wavelet Transform (DWT) lies in decomposing a signal into components with different frequency levels using Finite Impulse Response (FIR) filters, which consist of as following :

- A low-pass filter $h[n]$ to extract the approximation component.
- A high-pass filter $g[n]$ to extract the detail component.
- Downsampling after filtering, helps maintain data size and improve computational efficiency.

The wavelet decomposition process can be mathematically represented as follows[10,20,21]:

Low-frequency filtered signal (Approximation):

$$A_j[n] = \sum_k h\,[k-2n]A_{j-1}[k] \tag{7}$$

where $h[k]$ is the low-pass filter coefficient, and $A_{j-1}[k]$ represents the signal at the previous level.

High-frequency filtered signal (Details):

$$D_j[n] = \sum_k g\,[k-2n]A_{j-1}[k] \tag{8}$$

where $g[k]$ is the high-pass filter coefficient, and $D_j[n]$ represents the detail component at level j.

Reconstruction using the inverse DWT (IDWT):

$$A_{j-1}[n] = \sum_k h'\,[n-2k]A_j[k] + \sum_k g'\,[n-2k]D_j[k] \tag{9}$$

where $h'[n]$ and $g'[n]$ always represent the inverse filters used to reconstruct the original signal.

This mathematical formulation is the basis of wavelet analysis, which is used to decompose signals for various applications including data compression, pattern recognition, financial time-series analysis, and hydrological studies. It improves processing effectiveness and reduces noise in many digital applications .

4 Coiflets Wavelet

Coiflets wavelets, a type of wavelet developed by Ingrid Daubechies upon Ronald Coifman's suggestion in the spring of 1989, mark a crucial leap in the realm of wavelet theory, especially about signal processing. Coifman's proposal broadened the idea of vanishing moments to encompass both scaling and wavelet functions, moving away from the previous emphasis that was limited to just the wavelet function (Ψ). These are referred to as wavelets (Coif N), where (Coif) is a shorthand for (Coifman), and (N) signifies the candidate's rank. There exists a connection between the rank of the candidate and its length, denoted by (the length of the candidate = 6N), while the number of vanishing moments of the wavelet function (4) is expressed as (L = 2N), and for the scaling function (ϕ), the number of vanishing moments is (L_1 = 2N-1). The characteristics of the wavelet can be summarized as follows: Compact Support, Orthogonality, and approximate symmetry [22].

The mathematical expression for the Coiflets wavelet ψ (t) emerges from the mother wavelet using scaling and translation, and is formulated through the equation below:

$$\psi_{j,k}(t) = 2^{j/2}\,\psi(\;2^j\,t-k) \tag{10}$$

In this case, (j) denotes the scaling parameter, while (k) indicates the translation parameter.

The equation that follows illustrates how the denoised signal ($Y_{denoised}$)is reconstructed from wavelet coefficients and wavelet basis functions after implementing thresholding during the Coiflet wavelet denoising methodology. This equation can be expressed as:

$$Y_{denoised} = \sum\nolimits_{J,K} \widehat{W}_{j,k}\,\psi_{j,k}(t) \tag{11}$$

Here, $\widehat{W}_{j,k}$ represents the wavelet coefficients post-thresholding aimed at eliminating noise, and $\psi_{j,k}(t)$ consists of the wavelet basis functions employed in the process of reconstructing the signal.

Thus, Equation (11) denotes to signifies of the wavelet noise elimination or wavelet reconstruction formula after thresholding, highlighting the concept of wavelet denoising, whereby the signal undergoes processing to eliminate unintended noise while safeguarding its essential components. Consequently, wavelet denoising, especially through the use of Coiflets wavelets, is frequently utilized in signal processing and time series analysis because of its effectiveness in noise reduction and maintaining signal integrity [23].

5 Proposed Method

To create a VARMA model based on the discrete wavelet transformation coefficients of the Coiflets wavelet, specifically to create an approximation coefficients model (for low-pass filter) and detail coefficients model (for high-pass filter) for the Coiflets wavelet.

Suppose Y is a data matrix representing the m variables (y_1, y_2, …, y_m) and n observations. Perform a discrete wavelet transform for Coiflets wavelet of order (N = 1, 2, …, 5) for each variable (j = 1, 2, …, m) produce n/2+3N-1 if *n* is even and (n+1)/2+3N-1 if *n* is odd of the approximate and detailed coefficients (two partitions) after symmetrically extending for the observations y_j ($y_{j\{extended\}}$) as in the following formulas:

$$\mathrm{Aj}[\mathrm{n}] = \sum_{k=0}^{L-1} \mathrm{y}_{1\{\mathrm{extended}\}}[\mathrm{n}+k] \times h[k] \tag{12}$$

$$\mathrm{Dj}[\mathrm{n}] = \sum_{k=0}^{L-1} \mathrm{y}_{1\{\mathrm{extended}\}}[\mathrm{n}+k] \times g[k] \tag{13}$$

Where n varies over the appropriate indices of the data. Filter length (*L*), for Coiflets wavelet with order N = 1, *L* = 6 thus *h* (low-pass filter) and *g* (high-pass filter) respectively are:

$h[k]$ = [−0.0156557, −0.0727326, 0.3848648, 0.8525720, 0.3378977, −0.0727326]

$g[k]$ = [0.0727326, −0.3378977, 0.8525720, −0.3848648, 0.0727326, 0.0156557]

The number of coefficients for Coiflets wavelet N = 2, 3, 4, 5 is *L* = 9, 12, 15, 18.

equation 12 represents the approximation coefficients (first part), scale or father function Aj[n] and with n/2+3N-1 if n is even and (n+1)/2+3N-1 coefficients, which are proportional to the average observations.

In contrast, equation 13 represents (second part) the detail coefficients Dj[n], the mother or wavelet function with the same number of approximate coefficients, which are proportional to the differences (variance) of the observations.

Matrix of approximation and detail coefficients to A and D of the m variables as follows:

$$\mathrm{A} = [\mathrm{A1}[\mathrm{n}]\ \ \mathrm{A2}[\mathrm{n}] \dots\ \ \mathrm{Am}[\mathrm{n}]] \tag{14}$$

$$\mathrm{D} = [\mathrm{D1}[\mathrm{n}]\ \ \mathrm{D2}[\mathrm{n}] \dots\ \ \mathrm{Dm}[\mathrm{n}]] \tag{15}$$

Since direct estimation of VARMA models can be challenging, estimate a VAR (1) model as an approximation for the VARMA (1,1) model, depending on the A and D coefficients matrices. The VAR (1) model only includes the autoregressive component and is expressed as:

$$\mathrm{A}_t \approx c_1 + \Phi_1 \mathrm{A}_{t-1} + \eta_{1t} \tag{16}$$

Where c_1 is a constant vector, Φ_1 is the VAR (1) coefficient matrix, and η_{1t} is a white noise error vector at time *t* for the approximate coefficients model. The model parameters c_1 and Φ_1 are estimated from the approximate coefficient matrix A.

$$\mathrm{D}_t \approx c_2 + \Psi_1 \mathrm{D}_{t-1} + \eta_{2t} \tag{17}$$

Where c_2 is a constant vector, Ψ_1 is the VAR (1) coefficient matrix, and η_{2t} is a white noise error vector at time *t* for the detail coefficients model. The model parameters c_2 and Ψ_1 are estimated from the detail coefficient matrix D.

From equations 16 and 17 computing fitted values for the approximation (low pass filter) and detail (high pass filter) coefficients to obtain the estimated coefficient matrices:

$$\hat{\mathrm{A}} = [\hat{\mathrm{A}}1[\mathrm{n}]\ \ \hat{\mathrm{A}}2[\mathrm{n}] \dots\ \ \hat{\mathrm{A}}\mathrm{m}[\mathrm{n}]] \tag{18}$$

$$\hat{\mathrm{D}} = [\hat{\mathrm{D}}1[\mathrm{n}]\ \ \hat{\mathrm{D}}2[\mathrm{n}] \dots\ \ \hat{\mathrm{D}}\mathrm{m}[\mathrm{n}]] \tag{19}$$

Compute the inverse discrete wavelet transform (IDWT) from approximation and detail estimated coefficients $\hat{\mathrm{A}}\mathrm{j}[\mathrm{n}]$ and $\hat{\mathrm{D}}\mathrm{j}[\mathrm{n}]$ for j = 1, 2, …, m. If the DWT decomposes real data into low-pass filter (approximation) and high-pass filter (detail) components, the IDWT uses these components to reconstruct the original data as accurately as possible. The IDWT $\hat{\mathrm{A}}\mathrm{j}[\mathrm{n}]$ and $\hat{\mathrm{D}}\mathrm{j}[\mathrm{n}]$ are:

$$\hat{\mathrm{A}}\mathrm{j}[\mathrm{n}] = \sum_{k=0}^{L-1} \mathrm{Aj}\left[\frac{\mathrm{n}-k}{2}\right] \times h[k] \tag{20}$$

$$\hat{\mathrm{D}}\mathrm{j}[\mathrm{n}] = \sum_{k=0}^{L-1} \mathrm{Dj}\left[\frac{\mathrm{n}-k}{2}\right] \times h[k] \tag{21}$$

The reconstructed time series data Yj[n] is obtained by summing the two filtered data:

$$\mathrm{Yj}[\mathrm{n}] = \hat{\mathrm{A}}\mathrm{j}[\mathrm{n}] + \hat{\mathrm{D}}\mathrm{j}[\mathrm{n}] \tag{22}$$

For j = 1, 2, …, m, filtered data matrix YW is

$$\mathrm{YW} = [\mathrm{Y1}[\mathrm{n}]\ \ \mathrm{Y2}[\mathrm{n}] \dots\ \mathrm{Ym}[\mathrm{n}]] \tag{23}$$

The VAR (1) model for the YW matrix is expressed as:

$$\mathrm{YW}_t \approx c + \Phi\mathrm{W}_1 \mathrm{YW}_{t-1} + \eta_t \tag{24}$$

Where c is a constant vector, $\Phi\mathrm{W}_1$ is the VAR (1) coefficient matrix, and η_t is a white noise error vector, at time *t* for the YW matrix. The model parameters c and $\Phi\mathrm{W}_1$ are estimated from the YW matrix.

6 Simulation Study

VARMA models were simulated through two cases, the first case VARMA (1, 1) and the second case VARMA (2, 1) as follows:

Case 1: VARMA (1, 1)

- To simulate the Two-Dimensional VARMA(1,1) model, data were generated from a normal distribution using the parameter values presented in Table 1.

The constant values vector of the model (0.5; 0.2), the AR matrix represents the model's parameters for first-order autoregressive, and the MR matrix represents the model's parameters for the first-order moving average. White noise has a normal distribution with a zero mean vector and a standard deviation matrix (Sigma). The wavelet analysis of two variables and 10 observations of the first-order Coiflets wavelet is shown in Figure 1.

Table 1: Assumed Values for a Simulated Two-Dimensional (k = 2) VARMA (1, 1) Model.

Constant	AR		MR		Mean	Sigma	
0.5	0.5	0.1	0.3	0.1	0	1	0.3
0.2	0.2	0.3	0.1	0.2	0	0.3	1

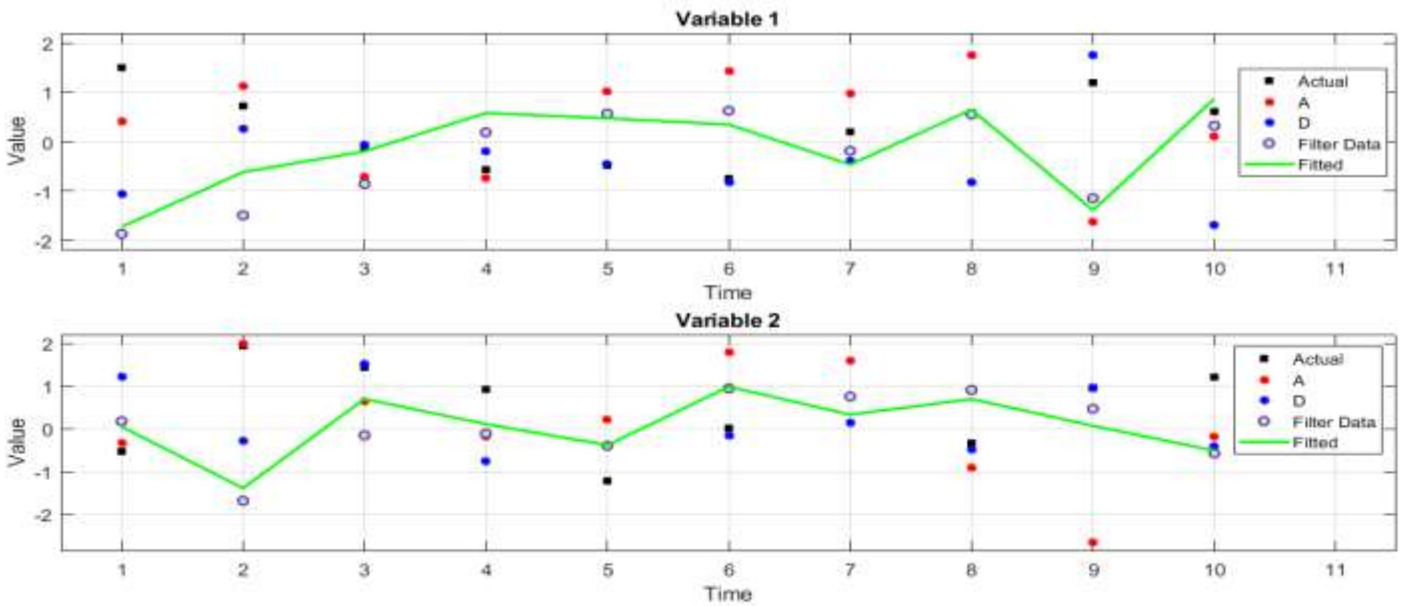

Figure 1: The wavelet analysis of two variables and 10 observations of the Coiflets1 wavelet.

The figure above demonstrates the application of first-order Coiflets wavelets (Coiflets1) to analyze two variables across ten observations. The initial signals were broken down into approximation (A) and detail (D) components, facilitating the distinction between long-term trends and short-term variations. Noise reduction was achieved through filtering techniques, resulting in filtered signals (Filter Data) that are clearer and more representative of the original data. The green line illustrates the fitted values generated using the VARMA model, showing a high degree of consistency with the actual observed values. This study highlights the important function of wavelet methods in improving data quality for forecasting models by effectively diminishing noise and enhancing the dependability of the resulting estimates.

The classical and proposed methods were applied to the first experimental dataset of the Two-Dimensional (k = 2) simulation with a sample size of n = 100, and the results are presented in Table 2.

Table 2 presents the simulation results of a Two-Dimensional (k = 2) VAR(1) model with a sample size of $n = 100$, comparing the classical method with wavelet-based approaches using Coiflets wavelets (Coiflets1 to Coiflets5). The classical method demonstrates higher values of MSE (1.0826), AIC (561.398), and BIC (576.968), compared to the wavelet-based methods. Among these, the Coiflets1 wavelet achieves the lowest MSE (0.2631), AIC *538.759), and BIC (*554.208), as denoted by the asterisks (*) and highlighted in yellow, indicating superior performance in noise reduction and model accuracy. In contrast, higher-order Coiflets wavelets show a slight increase in MSE, AIC, and BIC values, reaffirming Coiflets 2as the most effective method. These findings underscore the significant role of wavelets in enhancing predictive models by striking a balance between noise reduction and preserving data fidelity.

Table 2: Estimated Values for a Simulated Two-Dimensional (k = 2) VAR(1) Model in the First Experiment.

Method	Constant	AR		Mean (Y)	Sigma		MSE	AIC	BIC
Classical	0.3271	0.6041	0.1315	0.9063	0.9917	0.2032	1.0826	561.398	576.968
	0.1275	0.1047	0.5055	0.4266	0.2032	0.9304			
Coiflets1	0.0053	0.4699	-0.0219	-0.0015	0.9429	0.2697	0.2637	541.336	556.784
	-0.0095	0.1829	0.3379	0.0068	0.2697	0.9294			
Coiflets2*	0.0027	0.4779	-0.0155	-0.0053	0.9366	0.2550	*0.2631	*538.759	*554.208
	-0.0272	0.1836	0.3741	-0.0232	0.2550	0.9049			
Coiflets3	0.0345	0.4706	0.0097	0.0533	0.9397	0.2540	0.2698	539.631	555.079
	-0.0334	0.1662	0.3908	-0.0171	0.2540	0.9090			
Coiflets4	0.0391	0.4709	0.0148	0.0603	0.9415	0.2501	0.2771	540.384	555.832
	-0.0565	0.1722	0.3906	-0.0527	0.2501	0.9116			
Coiflets5	0.0178	0.4790	0.0415	0.0181	0.9250	0.2498	0.2848	539.845	555.294
	-0.0295	0.1350	0.4110	-0.0212	0.2498	0.9230			

An asterisk (*) denotes the optimal wavelet value corresponding to the minimum MSE, AIC, and BIC values.

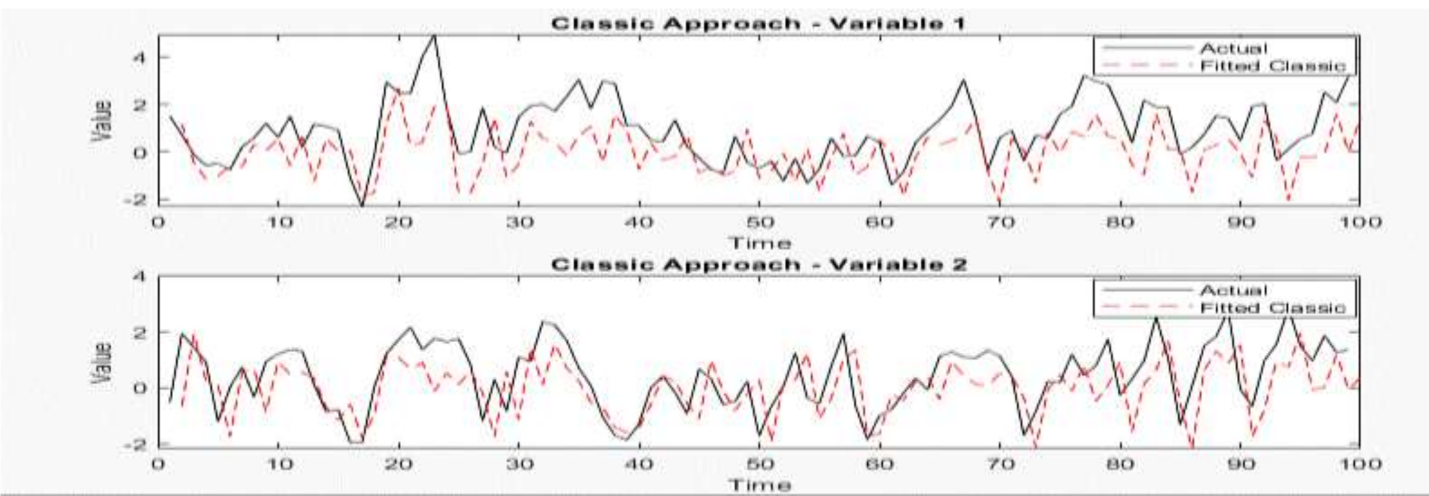

Figure 2: Actual and Fitted for Classical VAR (1) Model (First Experiment Study).

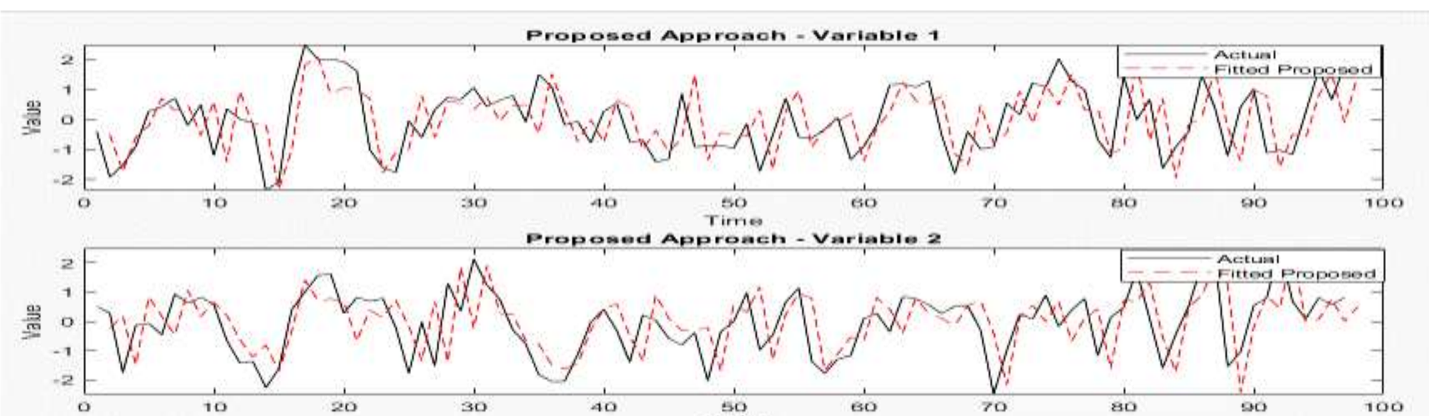

Figure 3: Actual and Fitted for Coiflets 2VAR (1) Model (First Experiment Study).

The figures compare the actual and fitted values for the VAR(1) model using the classical approach (Figure 2) and the Coiflets 2 wavelet-based method (Figure 3) over 100 observations. While the classical model shows limited capability in capturing dynamic variations, the Coiflets 2model demonstrates a stronger alignment with the actual values, particularly in identifying long-term trends and short-term fluctuations. Figure 3 highlights the superiority of the Coiflets 2model in reducing noise and improving prediction accuracy compared to the classical approach, emphasizing the importance of wavelets in enhancing the performance and reliability of predictive models.

The average results of repeating the simulation experiments (1000) times are shown in Table 3.

Table 3: Average of Estimated Values for a Simulated Two-Dimensional (k = 2) VAR(1) Mode.

Method	Sample size	Constant	AR		MSE	AIC	BIC
Classical	100	0.3597	0.6106	0.1308	1.6410	353.1933	379.2450
		0.1301	0.2116	0.4083			
Coiflets1*		0.0055	0.4490	0.0984	*0.3237	*199.3265	*225.3782
		0.0021	0.1439	0.3102			
Coiflets2		0.0115	0.4664	0.1002	0.3509	216.5083	242.5600
		0.0053	0.1505	0.3204			
Coiflets3		0.0117	0.4729	0.1002	0.3625	224.4072	250.4589
		0.0060	0.1528	0.3247			
Coiflets4		0.0121	0.4769	0.0995	0.3699	228.4470	254.4987
		0.0056	0.1538	0.3267			
Coiflets5		0.0124	0.4788	0.0991	0.3743	231.6761	257.7278
		0.0057	0.1541	0.3287			
Classical	500	0.3322	0.6369	0.1269	1.6552	1880.2	1922.4
		0.1099	0.2112	0.4297			
Coiflets1*		0.0010	0.4573	0.0941	*0.3271	*1100.1	*1142.3
		0.0002	0.1390	0.3196			
Coiflets2		0.0024	0.4730	0.0983	0.3529	1170.6	1212.7
		0.0009	0.1444	0.3300			
Coiflets3		0.0024	0.4789	0.0991	0.3632	1200.5	1242.7
		0.0011	0.1457	0.3344			
Coiflets4		0.0025	0.4820	0.0990	0.3687	1216.6	1258.8
		0.0011	0.1465	0.3366			
Coiflets5		0.0023	0.4838	0.0989	0.3719	1226.2	1268.3
		0.0011	0.1469	0.3379			
Classical	1000	0.3273	0.6402	0.1253	1.6458	3790.9	3840.0
		0.1083	0.2104	0.4329			
Coiflets1*		0.0005	0.4586	0.0927	*0.3273	*2215.8	*2264.9
		0.0002	0.1390	0.3207			
Coiflets2		0.0012	0.4740	0.0970	0.3530	2354.8	2403.8
		0.0006	0.1441	0.3311			
Coiflets3		0.0012	0.4797	0.0980	0.3632	2414.1	2463.1
		0.0007	0.1452	0.3356			
Coiflets4		0.0011	0.4826	0.0983	0.3685	2446.3	2.495.4
		0.0007	0.1456	0.3380			
Coiflets5		0.0011	0.4844	0.0981	0.3717	2466.2	2515.3
		0.0007	0.1459	0.3394			

An asterisk (*) denotes the optimal wavelet value corresponding to the minimum MSE, AIC, and BIC values.

Table 3 presents the average results from 1,000 simulation experiments conducted for a Two-Dimensional (k = 2) VAR(1) model using sample sizes of 100, 500, and 1,000. The classical method consistently records higher values for MSE, AIC, and BIC compared to wavelet-based methods, indicating less effective performance. Among the wavelet approaches, Coiflets1 consistently achieves the lowest MSE, AIC, and BIC values across all sample sizes, as denoted by the asterisks (*) and highlighted in yellow, demonstrating its superior ability to reduce noise and improve model accuracy. In contrast, higher-order wavelets (Coiflets2 to Coiflets5) show a slight increase in these metrics, further emphasizing the effectiveness of Coiflets1. These results underscore the pivotal role of Coiflets1 in improving predictive accuracy by achieving a balance between minimizing noise and preserving the integrity of the data.

- To simulate the Three -Dimensional VARMA(1,1) model, data were generated from a normal distribution using the parameter values presented in Table 4.

Table 4: Assumed Values for a Simulated Three-Dimensional (k = 3) VARMA(1,1) Mode.

Constant	AR			MR			Mean	Sigma		
0.5	0.5	0.1	0.4	0.3	0.1	0.2	0	1	0.3	0.2
0.2	0.2	0.3	0.2	0.1	0.2	0.3	0	0.3	1	0.2
0.3	0.4	0.2	0.1	0.2	0.4	0.3	0	0.2	0.2	1

The average results of repeating the simulation experiments (1000) times are shown in Table 5.

Table 5: Average Estimated Values for a Simulated Three-Dimensional (k = 3) VAR(1) Model.

Method	Sample size	Constant	AR			MSE	AIC	BIC
Classical	100	0.4118	0.5118	0.1074	0.4254	8.5077	508.7919	563.5005
		0.1525	0.0967	0.3658	0.3199			
		0.2103	0.2836	0.4331	0.1269			
Coiflets1*		0.0077	0.3953	0.0286	0.2452	*0.6272	203.8538	258.5623
		0.0029	0.0841	0.2455	0.2219			
		0.0028	0.2688	0.2890	0.0732			
Coiflets2		0.0208	0.4003	0.0343	0.2675	0.6912	234.8171	289.5257
		0.0099	0.0774	0.2558	0.2448			
		0.0079	0.2713	0.2948	0.0933			
Coiflets3		0.0252	0.3958	0.0394	0.2804	0.7176	247.3602	302.0688
		0.0125	0.0694	0.2626	0.2541			
		0.0105	0.2649	0.3016	0.1016			
Coiflets4		0.0289	0.3898	0.0454	0.2899	0.7348	255.0693	309.7779
		0.0138	0.0622	0.2690	0.2584			
		0.0124	0.2565	0.3097	0.1069			
Coiflets5		0.0290	0.3832	0.0507	0.2984	0.7472	259.9761	314.6847
		0.0146	0.0557	0.2741	0.2618			
		0.0126	0.2477	0.3175	0.1109			
Classical	500	0.3484	0.5360	0.0983	0.4250	8.8577	2666.9	2755.5
		0.1099	0.1001	0.3842	0.3201			
		0.1510	0.2873	0.4333	0.1413			
Coiflets1*		0.0009	0.4031	0.0277	0.2452	*0.6436	* 1154.2	* 1242.7
		0.0002	0.0794	0.2582	0.2167			
		0.0003	0.2562	0.2937	0.0858			
Coiflets2		0.0038	0.4112	0.0308	0.2634	0.7053	1273.0	1361.5
		0.0017	0.0754	0.2643	0.2405			
		0.0015	0.2657	0.2931	0.1036			
Coiflets3		0.0050	0.4122	0.0317	0.2702	0.7256	1322.9	1411.4
		0.0022	0.0711	0.2672	0.2492			
		0.0021	0.2667	0.2937	0.1088			
Coiflets4		0.0061	0.4116	0.0323	0.2743	0.7353	1349.7	1438.2
		0.0026	0.0680	0.2691	0.2538			
		0.0024	0.2660	0.2945	0.1115			
Coiflets5		0.0061	0.4103	0.0333	0.2773	0.7410	1366.0	1454.5
		0.0027	0.0654	0.2709	0.2565			
		0.0025	0.2645	0.2959	0.1132			
Classical	1000	0.3422	0.5386	0.0981	0.4240	8.9253	5357.2	5460.2
		0.1009	0.1004	0.3860	0.3215			

		0.1475	0.2890	0.4312	0.1428			
Coiflets1*		0.0005	0.4047	0.0300	0.2423	*0.6468	* 2329.7	* 2432.8
		0.0001	0.0801	0.2608	0.2146			
		0.0002	0.2568	0.2940	0.0847			
Coiflets2		0.0019	0.4134	0.0326	0.2596	0.7085	2553.1	2656.2
		0.0009	0.0764	0.2664	0.2385			
		0.0008	0.2669	0.2929	0.1024			
Coiflets3		0.0026	0.4149	0.0329	0.2657	0.7285	2647.8	2750.9
		0.0013	0.0726	0.2688	0.2473			
		0.0011	0.2687	0.2926	0.1076			
Coiflets4		0.0031	0.4151	0.0329	0.2689	0.7377	2699.2	2802.2
		0.0015	0.0699	0.2702	0.2518			
		0.0011	0.2690	0.2927	0.1099			
Coiflets5		0.0032	0.4148	0.0330	0.2711	0.7430	2731.1	2834.2
		0.0016	0.0680	0.2712	0.2545			
		0.0012	0.2685	0.2930	0.1114			

An asterisk (*) denotes the optimal wavelet value corresponding to the minimum MSE, AIC, and BIC values.

Table 5 presents the average results from 1,000 simulation experiments conducted for a Three-Dimensional (k = 3) VAR(1) model using sample sizes of 100, 500, and 1,000 . The classical method consistently records higher values for MSE, AIC, and BIC compared to wavelet-based methods, indicating less effective performance. Among the wavelet approaches, Coiflets1 consistently achieves the lowest MSE, AIC, and BIC values across all sample sizes, as denoted by the asterisks (*) and highlighted in yellow, demonstrating its superior ability to reduce noise and improve model accuracy. In contrast, higher-order wavelets (Coiflets2 to Coiflets5) show a slight increase in these metrics, further emphasizing the effectiveness of Coiflets1. These results underscore the pivotal role of Coiflets1 in improving predictive accuracy by achieving a balance between minimizing noise and preserving the integrity of the data.

Case 2: VARMA (2, 1)

- To simulate the Two-Dimensional VARMA(2,1) model, data were generated from a normal distribution using the parameter values presented in Table 6.

Table 6: Assumed Values for a Simulated Two-Dimensional (k = 2) VARMA (2, 1) Model.

Constant	AR1		AR2		MR		Mean	Sigma	
0.5	0.5	0.1	0.1	0.05	0.3	0.1	0	1	0.3
0.2	0.2	0.3	0.05	0.1	0.1	0.2	0	0.3	1

The constant values vector of the model (0.5; 0.2), the AR1 matrix represents the model's parameters for first-order autoregressive and the AR2 matrix represents the model's parameters for second-order autoregressive, and the MR matrix represents the model's parameters for the first-order moving average. White noise has a normal distribution with a zero mean vector and a standard deviation matrix (Sigma).

The classical and proposed methods were applied to the first experimental dataset of the Two-Dimensional (k = 2) simulation with a sample size of n = 100, and the results are presented in Table 7.

Table 7: Estimated Values for a Simulated Two-Dimensional (k = 2) VAR(2) Model in the First Experiment.

Method	Constant	AR1		AR2		Mean (Y)	Sigma		MSE	AIC	BIC
Classical	0.3081	0.7179	0.0873	-0.0830	0.1620	1.2171	0.9462	0.2234	1.9549	499.7678	536.2402
	0.2072	0.1566	0.6192	-0.0320	-0.1126	0.6941	0.2234	0.8793			
Coiflets1*	0.0308	0.4166	-0.0150	-0.1662	0.0422	0.0068	0.9412	0.2785	*0.1631	*200.6783	*237.1507
	-0.0026	0.1200	0.3603	-0.1082	-0.0725	-0.0297	0.2785	0.9076			

Coiflets2	0.0527	0.4362	0.0294	-0.1893	0.0243	0.0367	0.9216	0.2536	0.2034	233.0398	269.5122
	0.0037	0.1309	0.4213	-0.1399	-0.0934	-0.0209	0.2536	0.8730			
Coiflets3	0.0835	0.4306	0.0413	-0.2099	0.0181	0.0727	0.9292	0.2460	0.2244	244.7040	281.1764
	-0.0182	0.1426	0.4536	-0.1309	-0.1163	-0.0538	0.2460	0.8571			
Coiflets4	0.0983	0.4407	0.0349	-0.2218	0.0188	0.0921	0.9290	0.2383	0.2444	259.4693	295.9417
	-0.0239	0.1656	0.4732	-0.1272	-0.1496	-0.0574	0.2383	0.8446			
Coiflets5	0.0951	0.4530	0.0402	-0.2186	-0.0073	0.0908	0.9171	0.3251	0.2560	267.6396	304.1119
	-0.0235	0.1655	0.4853	-0.1464	-0.1421	-0.0606	0.2351	0.8413			

An asterisk (*) denotes the optimal wavelet value corresponding to the minimum MSE, AIC, and BIC values.

Table 7 presents the simulation results of a Two-Dimensional (k = 2) VAR(2) model with a sample size of $n = 100$, comparing the classical method with wavelet-based approaches using Coiflets wavelets (Coiflets1 to Coiflets5). The classical method demonstrates higher values of MSE (1.9549), AIC (499.7678), and BIC (536.2402), compared to the wavelet-based methods. Among these, the Coiflets1 wavelet achieves the lowest MSE (*0.1631), AIC (*200.6783), and BIC (* 237.1507), as denoted by the asterisks (*) and highlighted in yellow, indicating superior performance in noise reduction and model accuracy. In contrast, higher-order Coiflets wavelets show a slight increase in MSE, AIC, and BIC values, reaffirming Coiflets1 as the most effective method. These findings underscore the significant role of wavelets in enhancing predictive models by striking a balance between noise reduction and preserving data fidelity.

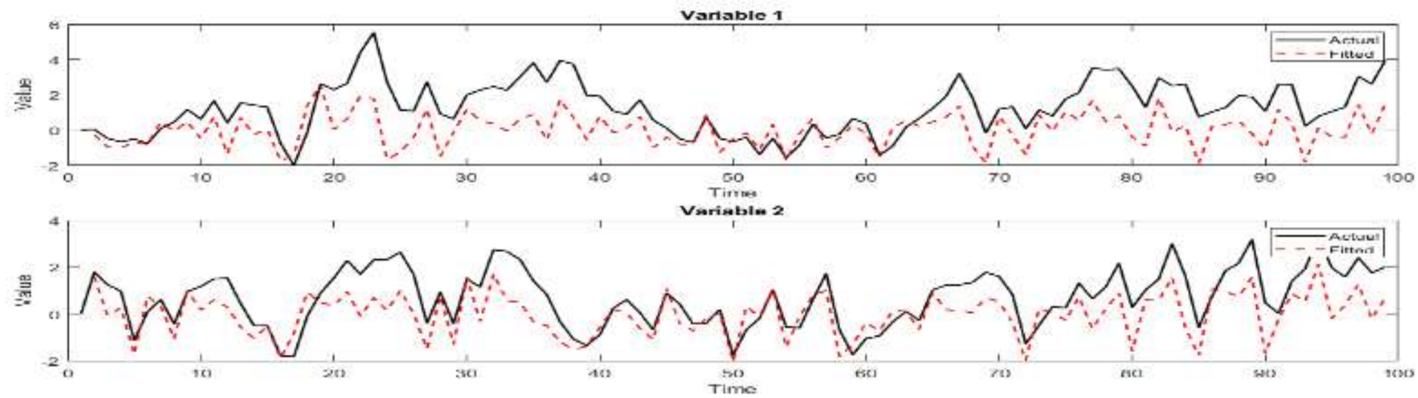

Figure 4: Actual and Fitted for Classical VAR (2) Model (First Experiment Study).

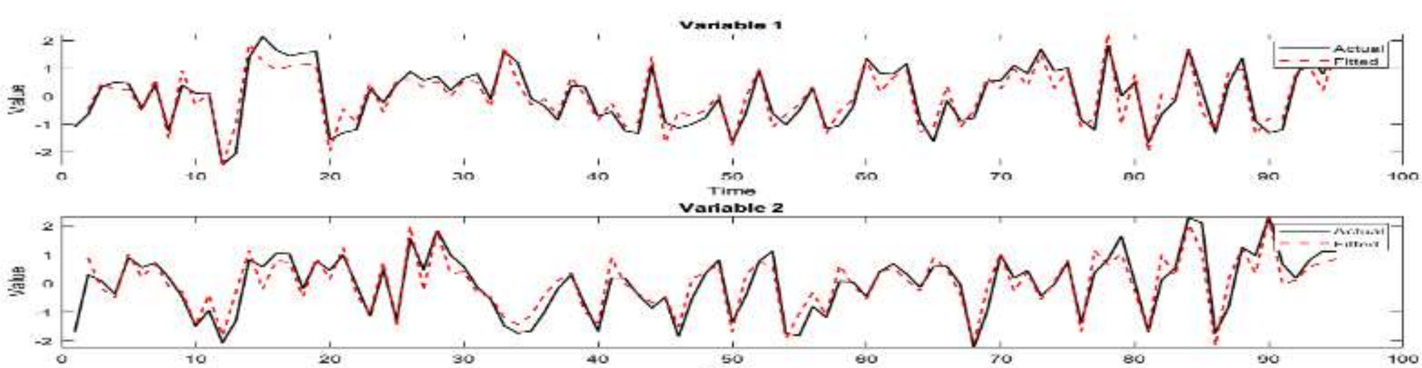

Figure 5: Actual and Fitted for Coiflets1 VAR (2) Model (First Experiment Study).

The figures compare the actual and fitted values for the VAR(2) model using the classical approach (Figure 4) and the Coiflets1 wavelet-based method (Figure 5) over 100 observations. While the classical model shows limited capability in capturing dynamic variations, the Coiflets1 model demonstrates a stronger alignment with the actual values, particularly in identifying long-term trends and short-term fluctuations. Figure 5 highlights the superiority of the Coiflets1 model in reducing noise and improving prediction accuracy compared to the classical approach, emphasizing the importance of wavelets in enhancing the performance and reliability of predictive models.

The average results of repeating the simulation experiments (1000) times are shown in Table 8.

Table 8: Average Estimated Values for a Simulated Two-Dimensional (k = 2) VAR(2) Model.

Method	Sample size	Constant	AR1		AR2		MSE	AIC	BIC
Classical	100	0.4531	0.7627	0.2015	-0.1457	-0.0380	3.1060	435.0913	471.5637
		0.1884	0.2973	0.4720	-0.0621	-0.0323			
Coiflets1*		0.0055	0.5041	0.1178	-0.2284	-0.0705	*0.2958	*193.5247	*229.9971
		0.0038	0.1714	0.3296	-0.1041	-0.1172			
Coiflets2		0.0234	0.5532	0.1367	-0.2648	-0.0871	0.3503	222.4497	258.9221
		0.0130	0.1964	0.3546	-0.1247	-0.1303			
Coiflets3		0.0324	0.5729	0.1416	-0.2776	-0.0941	0.3767	236.6374	273.1098
		0.0195	0.2065	0.3636	-0.1325	-0.1366			
Coiflets4		0.0345	0.5823	0.1421	-0.2844	-0.0972	0.3905	244.6016	281.0740
		0.0214	0.2119	0.3673	-0.1357	-0.1396			
Coiflets5		0.0355	0.5872	0.1429	-0.2870	-0.0994	0.3989	248.9367	285.4091
		0.0219	0.2148	0.3696	-0.1375	-0.1411			
Classical	500	0.4018	0.7894	0.1989	-0.1326	-0.0420	3.1850	2206.1	2265.1
		0.1494	0.2979	0.4935	-0.0647	-0.0158			
Coiflets1*		0.0006	0.5190	0.1145	-0.2271	-0.0708	*0.3160	*1080.2	*1139.2
		0.0002	0.1733	0.3405	-0.1090	-0.1140			
Coiflets2		0.0048	0.5649	0.1331	-0.2618	-0.0862	0.3689	1195.9	1254.9
		0.0023	0.1980	0.3624	-0.1297	-0.1274			
Coiflets3		0.0066	0.5826	0.1389	-0.2754	-0.0920	0.3905	1243.6	1302.6
		0.0038	0.2065	0.3714	-0.1371	-0.1334			
Coiflets4		0.0074	0.5917	0.1416	-0.2822	-0.0952	0.4019	1268.9	1327.9
		0.0045	0.2107	0.3761	-0.1406	-0.1367			
Coiflets5		0.0074	0.5973	0.1431	-0.2864	-0.0970	0.4090	1284.6	1343.6
		0.0045	0.2131	0.3789	-0.1425	-0.1389			
Classical	1000	0.3934	0.7921	0.1967	-0.1308	-0.0408	3.1699	4425.3	4494.0
		0.1464	0.2971	0.4964	-0.0657	-0.0133			
Coiflets1*		0.0002	0.5209	0.1133	-0.2274	-0.0699	*0.3179	*2183.6	*2252.3
		0.0001	0.1727	0.3418	-0.1087	-0.1137			
Coiflets2		0.0023	0.5662	0.1324	-0.2617	-0.0856	0.3707	2410.9	2479.6
		0.0012	0.1969	0.3636	-0.1288	-0.1278			
Coiflets3		0.0033	0.5835	0.1385	-0.2750	-0.0913	0.3919	2504.1	2572.8
		0.0020	0.2049	0.3727	-0.1360	-0.1340			
Coiflets4		0.0036	0.5924	0.1413	-0.2820	-0.0942	0.4032	2554.0	2622.7
		0.0023	0.2088	0.3776	-0.1395	-0.1374			
Coiflets5		0.0036	0.5979	0.1429	-0.2861	-0.0960	0.4101	2585.4	2.654.1
		0.0023	0.2111	0.3806	-0.1417	-0.1394			

An asterisk (*) denotes the optimal wavelet value corresponding to the minimum MSE, AIC, and BIC values.

Table 8 presents the average results from 1,000 simulation experiments conducted for a Two-Dimensional (k = 2) VAR(2) model using sample sizes of 100, 500, and 1,000 . The classical method consistently records higher values for MSE, AIC, and BIC compared to wavelet-based methods, indicating less effective performance. Among the wavelet approaches, Coiflets1 consistently achieves the lowest MSE, AIC, and BIC values across all sample sizes, as denoted by the asterisks (*) and highlighted in yellow, demonstrating its superior ability to reduce noise and improve model accuracy. In contrast, higher-order wavelets (Coiflets2 to Coiflets5) show a slight increase in these metrics, further emphasizing the effectiveness of Coiflets1. These results underscore the pivotal role of Coiflets1 in improving predictive accuracy by achieving a balance between minimizing noise and preserving the integrity of the data.

- To simulate the Three-Dimensional VARMA(2,1) model, data were generated from a normal distribution using the parameter values presented in Table 9.

Table 9: Assumed Values for a Simulated Three-Dimensional (k = 3) VARMA(2,1) Model.

Constant	AR1			AR2			MR			Mean	Sigma		
0.5	0.5	0.1	0.2	0.10	0.05	0.02	-0.2	-0.1	-0.2	0	1	0.3	0.2
0.2	0.2	0.3	0.2	0.05	0.10	0.05	-0.1	-0.2	-0.1	0	0.3	1	0.2
0.1	0.2	0.2	0.1	0.30	0.02	0.01	-0.2	-0.1	-0.2	0	0.2	0.2	1

The average results of repeating the simulation experiments (1000) times are shown in Table 10.

Table 10: Average Estimated Values for a Simulated Three-Dimensional (k = 3) VAR(2) Model.

Method	n	Constant	AR1			AR2			MSE	AIC	BIC
Classical	100	0.9289	0.3276	0.0359	0.0829	0.2039	0.1306	0.0741	14.1045	466.5762	544.7313
		0.4822	0.1343	0.1083	0.1585	0.1543	0.1373	0.1116			
		0.4216	0.0521	0.1305	-0.0318	0.4394	0.0999	0.0435			
Coiflets1*		0.0129	0.1817	-0.0042	-0.0098	-0.0592	-0.0015	-0.0045	*0.0431	*-256.6773	*-178.5222
		0.0089	0.0379	0.0607	0.0611	-0.0158	-0.0228	-0.0056			
		0.0063	-0.0104	0.0653	-0.0871	-0.0058	-0.0033	-0.0297			
Coiflets2		0.0381	0.1956	-0.0024	-0.0222	-0.0768	-0.0136	-0.0328	0.0492	-210.1358	-131.9806
		0.0250	0.0425	0.0675	0.0551	-0.0264	-0.0319	-0.0181			
		0.0234	-0.0049	0.0717	-0.0930	-0.0054	-0.0121	-0.0469			
Coiflets3		0.0579	0.2061	-0.0009	-0.0289	-0.0738	-0.0190	-0.0518	0.0560	-174.5479	-96.3928
		0.0430	0.0473	0.0709	0.0526	-0.0292	-0.0366	-0.0276			
		0.0467	-0.0002	0.0730	-0.0954	0.0032	-0.0239	-0.0567			
Coiflets4		0.0733	0.2100	0.0005	-0.0316	-0.0708	-0.0193	-0.0677	0.0621	-142.5598	-64.4047
		0.0567	0.0503	0.0711	0.0518	-0.0282	-0.0401	-0.0334			
		0.0644	0.0006	0.0742	-0.0987	0.0150	-0.0334	-0.0682			
Coiflets5		0.0853	0.2126	0.0011	-0.0339	-0.0656	-0.0178	-0.0826	0.0668	-122.8869	-44.7318
		0.0678	0.0533	0.0703	0.0522	-0.0264	-0.0425	-0.0373			
		0.0778	0.0010	0.0731	-0.1011	0.0259	-0.0384	-0.0745			
Classical	500	0.9999	0.3386	0.0276	0.0746	0.2213	0.1203	0.0618	17.2502	1.6258	1.7522
		0.5635	0.1313	0.1204	0.1530	0.1401	0.1547	0.0997			
		0.5557	0.0409	0.1313	-0.0268	0.4322	0.0884	0.0459			
Coiflets1*		0.0025	0.1842	-0.0054	-0.0055	-0.0394	-0.0002	-0.0032	*0.0232	*-2258.2	*-2131.8
		0.0014	0.0386	0.0637	0.0667	-0.0111	-0.0110	-0.0013			
		0.0019	-0.0191	0.0706	-0.0800	-0.0035	-0.0012	-0.0155			
Coiflets2		0.0099	0.1927	0.0010	-0.0126	-0.0483	-0.0048	-0.0113	0.0250	-2062.2	-1935.8
		0.0059	0.0389	0.0693	0.0633	-0.0158	-0.0143	-0.0057			
		0.0069	-0.0146	0.0771	-0.0845	-0.0051	-0.0034	-0.0208			
Coiflets3		0.0156	0.1974	0.0032	-0.0176	-0.0510	-0.0072	-0.0178	0.0263	-1952.0	-1825.6
		0.0104	0.0391	0.0715	0.0611	-0.0178	-0.0167	-0.0095			
		0.0130	-0.0127	0.0797	-0.0877	-0.0044	-0.0071	-0.0251			
Coiflets4		0.0199	0.2000	0.0043	-0.0204	-0.0517	-0.0082	-0.0226	0.0272	-1887.3	-1760.9
		0.0142	0.0396	0.0725	0.0599	-0.0187	-0.0180	-0.0118			
		0.0176	-0.0114	0.0809	-0.0899	-0.0022	-0.0095	-0.0280			
Coiflets5		0.0230	0.2019	0.0047	-0.0225	-0.0513	-0.0084	-0.0269	0.0279	-1842.5	-1716.0
		0.0170	0.0402	0.0730	0.0590	-0.0186	-0.0190	-0.0138			
		0.0209	-0.0106	0.0815	-0.0914	0.0006	-0.0114	-0.0305			
Classical	1000	1.0201	0.3381	0.0280	0.0725	0.2226	0.1175	0.0621	17.6363	3034.3	3181.5
		0.5690	0.1296	0.1224	0.1521	0.1408	0.1553	0.0987			

		0.5933	0.0411	0.1289	-0.0275	0.4261	0.0879	0.0493			
Coiflets1*		0.0012	0.1847	-0.0052	-0.0070	-0.0375	0.0003	-0.0024	*0.0211	*-4390.9	*-4243.7
		0.0007	0.0393	0.0642	0.0661	-0.0105	-0.0103	-0.0010			
		0.0008	-0.0168	0.0692	-0.0799	-0.0037	-0.0008	-0.0144			
Coiflets2		0.0052	0.1930	0.0019	-0.0130	-0.0440	-0.0030	-0.0067	0.0229	-4072.6	-3925.3
		0.0030	0.0396	0.0698	0.0635	-0.0134	-0.0128	-0.0034			
		0.0036	-0.0121	0.0758	-0.0836	-0.0049	-0.0021	-0.0174			
Coiflets3		0.0083	0.1969	0.0044	-0.0172	-0.0466	-0.0047	-0.0102	0.0238	-3918.5	-3771.3
		0.0057	0.0393	0.0720	0.0616	-0.0149	-0.0146	-0.0055			
		0.0070	-0.0102	0.0787	-0.0864	-0.0050	-0.0040	-0.0202			
Coiflets4		0.0108	0.1993	0.0054	-0.0195	-0.0476	-0.0054	-0.0129	0.0244	-3833.6	-3686.3
		0.0078	0.0394	0.0730	0.0605	-0.0154	-0.0156	-0.0068			
		0.0096	-0.0092	0.0802	-0.0882	-0.0041	-0.0053	-0.0222			
Coiflets5		0.0126	0.2008	0.0061	-0.0214	-0.0479	-0.0057	-0.0156	0.0248	-3775.6	-3628.3
		0.0093	0.0395	0.0737	0.0597	-0.0157	-0.0161	-0.0078			
		0.0115	-0.0086	0.0812	-0.0895	-0.0028	-0.0063	-0.0237			

An asterisk (*) denotes the optimal wavelet value corresponding to the minimum MSE, AIC, and BIC values.

Table 10 presents the average results from 1,000 simulation experiments conducted for a Three-Dimensional (k = 3) VAR(2) model using sample sizes of 100, 500, and 1,000 . The classical method consistently records higher values for MSE, AIC, and BIC compared to wavelet-based methods, indicating less effective performance. Among the wavelet approaches, Coiflets1 consistently achieves the lowest MSE, AIC, and BIC values across all sample sizes, as denoted by the asterisks (*) and highlighted in yellow, demonstrating its superior ability to reduce noise and improve model accuracy. In contrast, higher-order wavelets (Coiflets2 to Coiflets5) show a slight increase in these metrics, further emphasizing the effectiveness of Coiflets1. These results underscore the pivotal role of Coiflets1 in improving predictive accuracy by achieving a balance between minimizing noise and preserving the integrity of the data.

7 Real Data

The graphs display the monthly time series data of the Iraqi economy, measured in billions of Iraqi dinars, covering the period from January 31, 2010, to July 31, 2024, with a total of 175 observations (n=175). The dataset include key indicators such as Official Reserves (OR), and Broad Money (M2) [24] , known as a time series plot (See Figures 6 and 7). The first stage in time series analysis is plotting the observations over time, which provides initial insights into the series' characteristics, such as its stability and the presence of trends or seasonal variations. This preliminary evaluation helps prepare for more accurate stability tests later on.

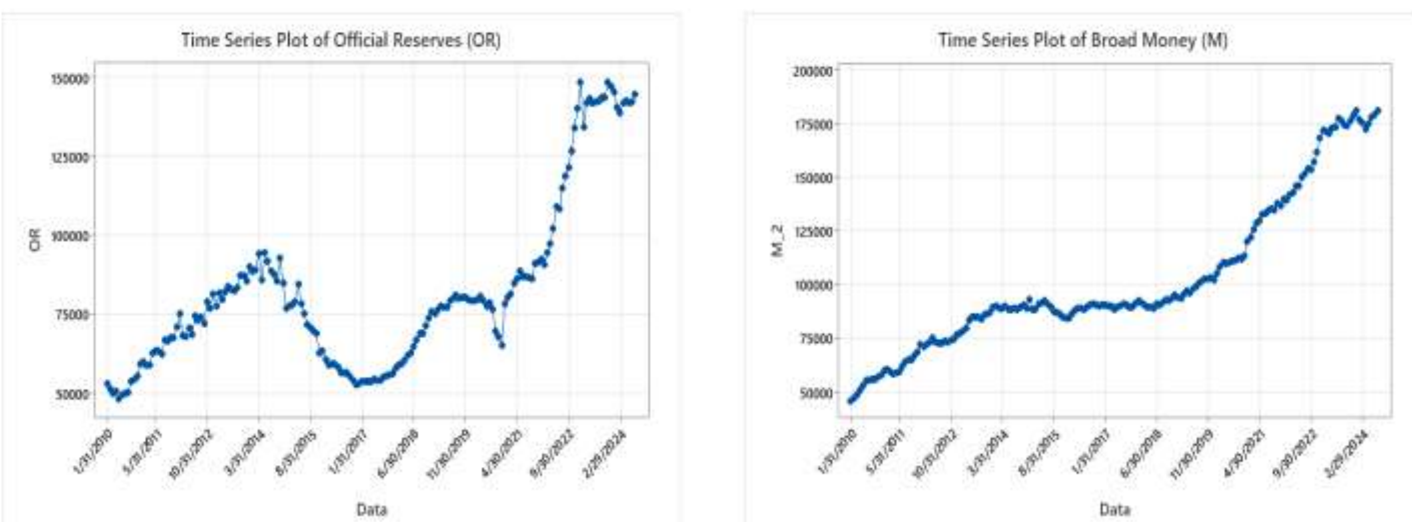

Figure 6: Time series plot for monthly financial and monetary dataset of the Iraqi economy (Billion IQD).

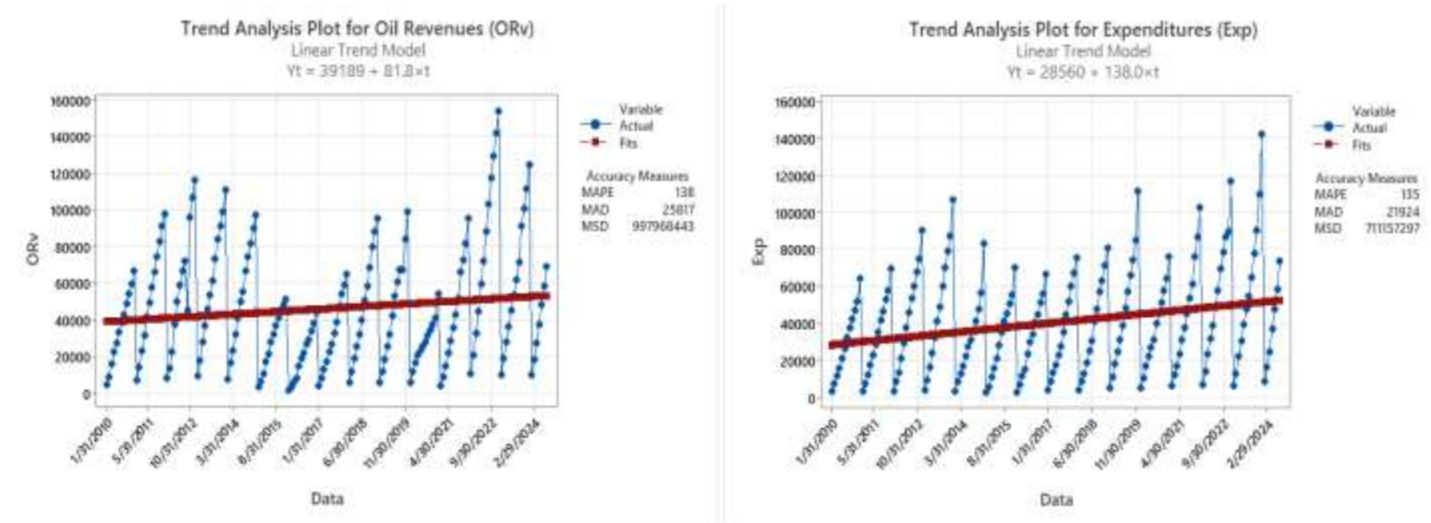

Figure 7: Analysis of the General Trend for monthly financial and monetary dataset of the Iraqi economy (Billion IQD).

The visual examination of Figures (1) and (2) reveals that all Two-time series display non-stationarity concerning both variance and mean, indicating a general upward trend over a span of 175 months.

To validate these findings, stationarity tests like the Augmented Dickey-Fuller (ADF) test should be used . If non-stationarity is identified, differencing methods, particularly first differences, can be employed to remove trends. Once stationarity is achieved in the variance and mean for all two time series, various analyses using different models, such as VAR modelling, can then be performed on the stationary data to ensure the accuracy and reliability of the results derived from this model.

Unit Root Test for Trend Detection

Following the logarithmic transformation (Ln) applied to the four-time series represented by the monthly financial and monetary dataset of Iraq's economy (in Billion IQD), the aim was to stabilize the variance across these series. Subsequently, specific assessments, like the Augmented Dickey-Fuller (ADF) tests, are utilized to check the stationary nature of the time series data about a mean value. These assessments are carried out with the optimal lag durations **(**Maxlag = 13, determined by the Schwarz information criterion (SIC)), as detailed in Table 11, to ascertain if a time series maintains stability around a mean or linear trend, or if it continues to show instability due to the existence of a unit root . The hypotheses under examination include the following:

• Null Hypothesis H0: The data is non-stationary and contains a unit root.
• Alternative Hypothesis H1: The data is trend stationary and lacks a unit root.

Table 11: Augmented Dickey-Fuller Test for monthly financial and monetary dataset of the Iraqi economy (Billion IQD).

Data	Case	Form	lag	Test Statistics	Critical Value	P-Value at 5%	Decision I(d)
Ln (M2)	level	Constant	0	-1.543287	-2.878015	0.5094	Non- Stationary I(0)
D1 (Ln (M2))	First Difference	Constant	0	-10.62810	-2.878113	0.0000*	stationary I(1)
Ln(OR)	level	None (Constant, Linear Trend)	0	1.983990	-1.942666	0.9888	Non- Stationary I(0)
D1 (Ln(OR))	First Difference	Constant	0	-14.42290	-2.878113	0.0000*	stationary I(1)

Stationary I(1) ***:** The null hypothesis is rejected ($|Test\ statistic\ | \geq |critical\ value|$) at significance level = 0.05. Or p-value ≤ 0.05.

Table 11 displays the results from the ADF stationarity examination, indicating that all variables (M2 and OR) are non-stationary in their original levels since the unit root null hypothesis remains unchallenged. Nevertheless, when these variables are differentiated for the first and second time, the null hypothesis is successfully rejected, signifying that the first differences of both variables are stationary and demonstrate the integration of the first order, I(1) for M2 and OR. The outcomes of the ADF stationarity examination are as follows:

• First Difference for Ln (M2) (Broad Money): The computed absolute test statistic is 10.62810, surpassing the critical absolute value of 2.878015 (the p-value is 0.0000, which is less than 0.05), indicating the rejection of the null hypothesis. Consequently, the first difference of Ln (M2) is stationary I(1).

• First Difference for Ln(OR) (Official Reserves): The computed absolute test statisti is 14.42290, which exceeds the absolute critical value of 2.878113 (the p-value is 0.0000, which is less than 0.05), signifying the rejection of the null hypothesis. Hence, the first difference of Ln(OR) is stationary (I(1)).

After confirming the stationarity of the time series, these results become crucial for constructing two-dimensional approximate VAR models, which require stationary data for accurate and interpretable outcomes. In this context, various noise removal methods can be applied to assess the impact of dependent variables, such as Broad Money (M2) and Official Reserves (OR), on the internal economic system.

The classical and proposed methods for constructing two-dimensional (k = 2) approximate VAR(p) models, with lag orders set to p = 1, p = 2, and p = 3, were applied to the stationary monthly time series data of the Iraqi economy.The results are presented in the following table .

Table 12: Estimated Values for Iraqi Monthly Economy Time Series Two-Dimensional (k = 2) VAR(1) Model.

Method	Constant	AR		MSE	AIC	BIC
Classical	-0.0079298	0.19702	0.05548	0.03691	966.979	985.899
	0.0062075	0.13699	-0.12715			
Coiflets1**	-0.031723	0.13104	0.099042	**0.03338	*920.467	*939.317
	0.015636	0.14875	-0.16805			
Coiflets2	-0.063639	0.13748	0.094526	0.035829	923.279	942.129
	0.027734	0.15795	-0.15971			
Coiflets3	-0.039987	0.1481	0.096059	0.037043	923.948	942.798
	0.020148	0.17088	-0.15542			
Coiflets4	-0.04151	0.15869	0.098648	0.039026	925.022	943.872
	0.024613	0.1741	-0.14946			
Coiflets5	-0.052889	0.1613	0.1045	0.036897	925.136	943.986
	0.015258	0.1547	-0.13318			

An asterisk (*) denotes the optimal wavelet value corresponding to the minimum AIC, and BIC values.

The results presented in this study provide a comprehensive evaluation of **VAR(*p*)** bivariate models with different lag orders (p = 1, 2, and 3) applied to Iraq's monthly economic data. The classical approach, which does not incorporate wavelet transformations, consistently exhibited higher Mean Squared Error (MSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) values across all lag orders. This indicates that traditional models had lower predictive accuracy and efficiency compared to wavelet-enhanced models.

After testing models with (p = 1, 2, and 3) lag orders, the analysis confirmed that the VAR(1) model using the Coiflets1 wavelet transformation achieved the best performance, recording the lowest MSE (0.03338), AIC (920.467), and BIC (939.317). These findings, summarized in Table 12, highlight the model's superior predictive capability while maintaining simplicity and reducing noise, as indicated by the asterisks (**) and highlighted in yellow.

However, upon examining the statistical significance of the parameters (T-statistics and P-values), it was observed that some parameters in the VAR(2) and VAR(3) models were not statistically significant, implying that increasing the lag order does not provide substantial predictive improvements. Instead, it could introduce unnecessary complexity without tangible benefits, leading to an overfitting problem.

Based on these findings, the VAR(1) model was identified as the optimal choice for modeling Iraq’s monthly economic data, striking the best balance between predictive accuracy and model simplicity.

The figures compare the actual and fitted values for the VAR (1) model using the classical approach (Figure 8) and the Coiflets1 wavelet method (Figure 9) across 175 observations. Figure 8 demonstrates an acceptable but limited performance of the classical approach, while Figure 9 highlights the superiority of the Coiflets1 wavelets in reducing noise and improving prediction accuracy, particularly during critical periods. These results highlight the importance of using wavelets to analyze dynamical systems with greater accuracy and reliability.

The figure above (Figure 10) demonstrates the application of first-order Coiflets wavelets (Coiflets1) to analyze two variables both Broad Money (M2) and Official Reserves (OR) across all observations. The initial signals were broken down into approximation (A) and detail (D) components, facilitating the distinction between long-term trends and short-term variations.

Filtering techniques are used to reduce noise to produce a filtered signal (filtered data) that closely matches the actual data.

The green line represents the fitted values obtained from the VARMA model, which show a strong agreement with the observed actual value. The results further emphasize the importance of wavelet techniques in forecasting models as they help to minimize noise, improve data quality, and increase the robustness of the analysis as well as the estimated predictions. Moreover, the results show that Coiflets1 wavelets can adequately capture the time-varying connections and patterns among the economic time series.

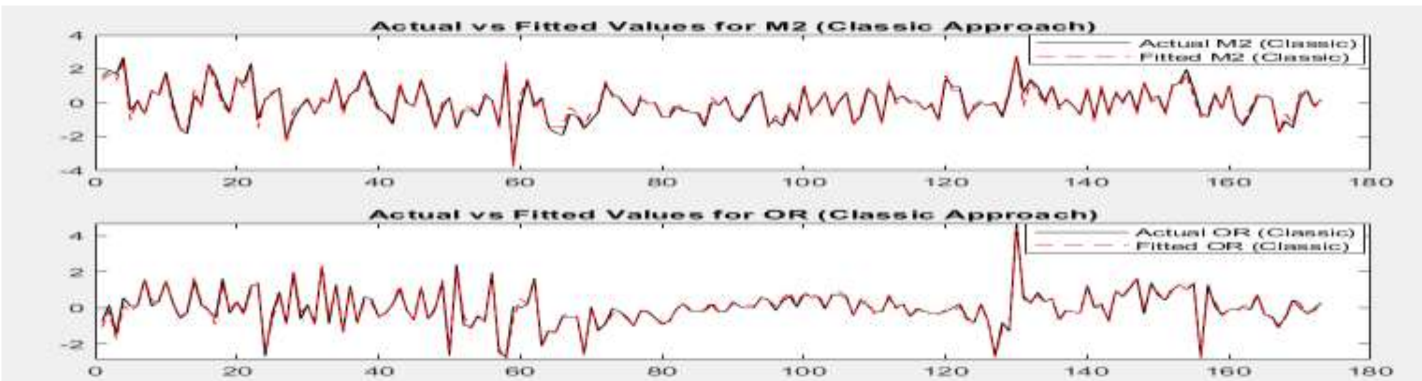

Figure 8: Actual and Fitted of Broad Money (M2) and Official Reserves (OR) for Classical VAR (1) Model.

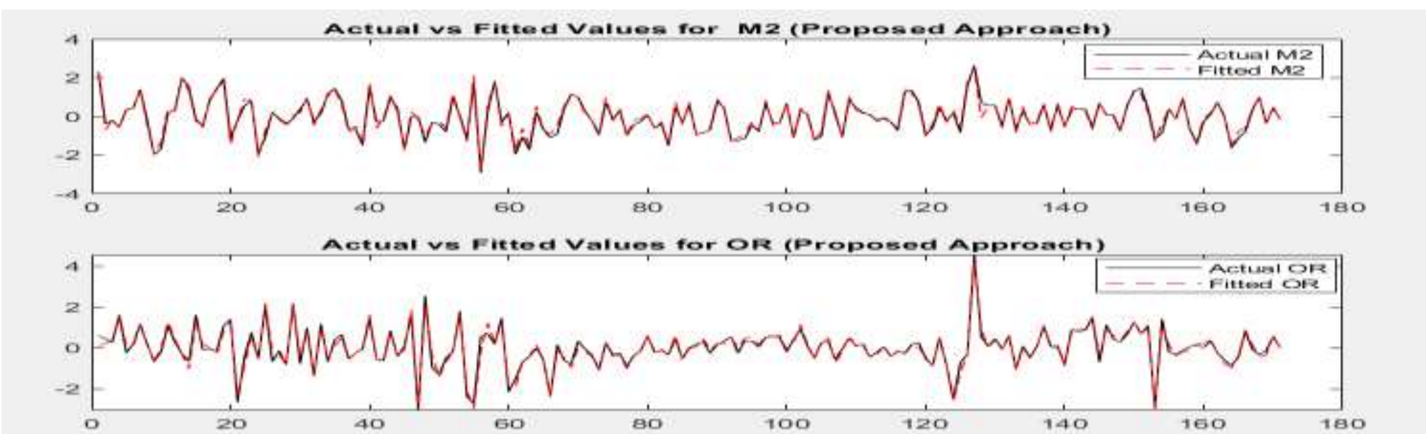

Figure 9: Actual and Fitted of Broad Money (M2) and Official Reserves (OR) for Coiflets1 VAR (1) Model.

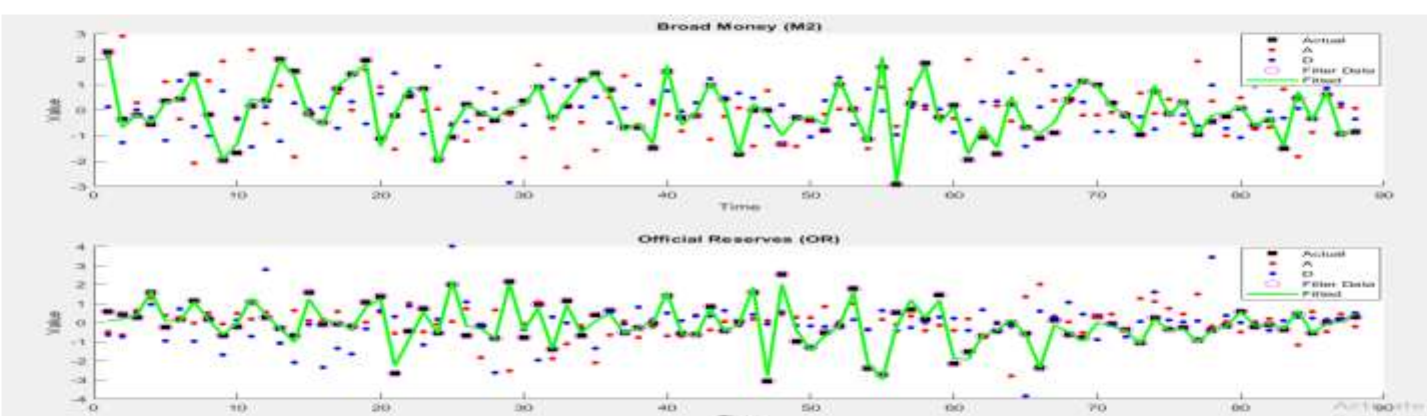

Figure 10: The wavelet analysis of Broad Money (M2) and Official Reserves (OR) of the Coiflets1 wavelet.

Proposed VAR(1) Model for Stationary Iraqi Monthly Economic Time Series Data

As defined in the theoretical framework, the proposed VAR(1) model, incorporating Coiflets1 wavelet transformations was selected as the optimal approach for analyzing the monthly economic time series data for Iraq. This approach presents a better performance in minimizing MSE, AIC, and BIC values compared to VAR(2) and VAR(3) models (see Table 12). By accurately capturing dynamic interactions between Broad Money (M2) and Official Reserves (OR), the model improves forecasting accuracy, while also ensuring the integrity of the original data and keeping the noise to a minimum. Adding Coiflets1 wavelet transforms further optimizes the model in data reconstruction and parameter estimation, making the model applicable to economic studies with strong analytical and forecasting capabilities. Estimation results for the model are shown in Table 13, with six estimated parameters (N = 6). The mathematical expression of this AR-stationary 2-dimensional VAR(1) model as expressed in equation below:

$$(1 - \Phi_1 B)Y_t = C + e_t \tag{25}$$

The expanded equation becomes, as follows:

$$\left(I - \begin{bmatrix} 0.13104 & 0.099042 \\ 0.14875 & -0.16805 \end{bmatrix} B\right) Y_t = \begin{bmatrix} -0.031723 \\ 0.015636 \end{bmatrix} + e_t$$

Where:

- Y_t is the vector of dependent variables at time t ($Y_t = [M2_t, OR_t]^T$).

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.13104 & 0.099042 \\ 0.14875 & -0.16805 \end{bmatrix} B\right) \begin{bmatrix} M2_t \\ OR_t \end{bmatrix} = \begin{bmatrix} -0.031723 \\ 0.015636 \end{bmatrix} + \begin{bmatrix} e_{M2_t} \\ e_{OR_t} \end{bmatrix}$$

Table 13: Presents the estimation results for the AR-Stationary 2-Dimensional VAR (1) Model.

Parameter	Value	Standard Error	t Statistic	P-Value
Constant(1)	-0.03172	0.069098	-0.4591	0.64616
Constant(2)	0.015636	0.072573	0.21546	0.82941
AR{1}(1,1)	0.13104	0.075857	1.7275	0.08408
AR{1}(2,1)	0.14875	0.079672	1.8671	0.061893
AR{1}(1,2)	0.099042	0.072653	1.3632	0.17282
AR{1}(2,2)	-0.16805	0.076307	-2.2023	0.027645

8 Discussion

Table 13 presents the estimation results for the AR-Stationary 2-Dimensional VAR(1) Model, highlighting the dynamic relationships between Broad Money (M2) and Official Reserves (OR). The results indicate the significance of the AR{1}(2,2) coefficient, with a p-value of 0.0276 (P-Value < 0.05), underscoring its key role in explaining the dynamic interactions between the variables. Conversely, the remaining coefficients, such as the constants and AR(1) terms, exhibit minimal statistical relevance.

The proposed model demonstrates its effectiveness by utilizing Coiflets1 wavelet transformations, which improves data integrity and reduces noise, making it a powerful tool for studying dynamic linkages within the Iraqi economy.

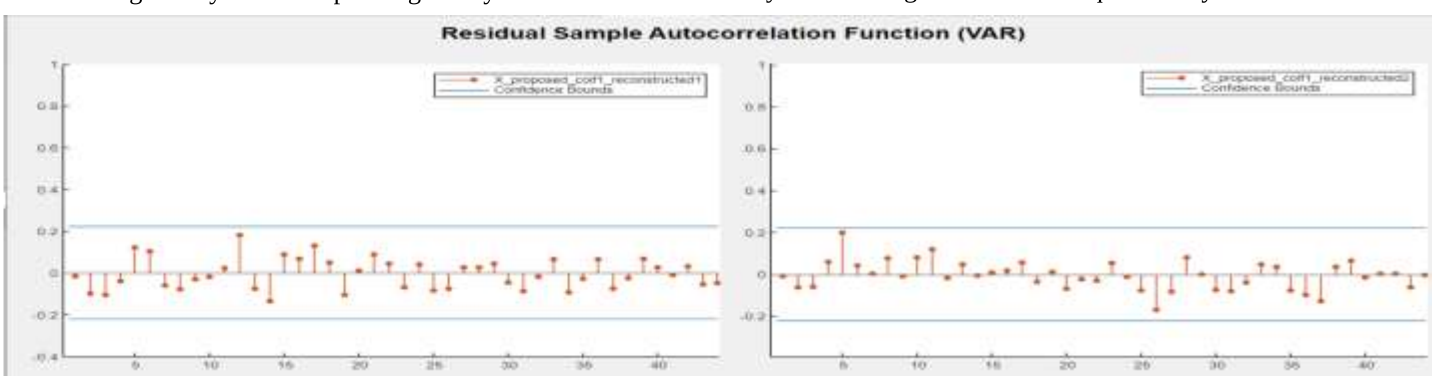

Figure 11: Residual Sample Autocorrelation Function the Proposed VAR (1) Model.

Figure 11 illustrates the residual autocorrelation function (ACF) for the proposed VAR(1) model, examining if any autocorrelation persists in the residuals after estimation. The results show all autocorrelation coefficients fall within the confidence bands (blue lines), meaning that no significant autocorrelation exists among the residuals.

Furthermore, the residuals for Broad Money (M2) and Official Reserves (OR) appear to be randomly scattered around zero which confirms that there is no autocorrelation present in the residuals, verifying the core underlying assumptions of the linear model. As a result, the model is robust, thereby confirming the ability of the model to accurately capture the evolving inter-relationships between economic variables without introducing bias or structure in the residuals, hence its effectiveness for economic forecasting andtime series analysis.

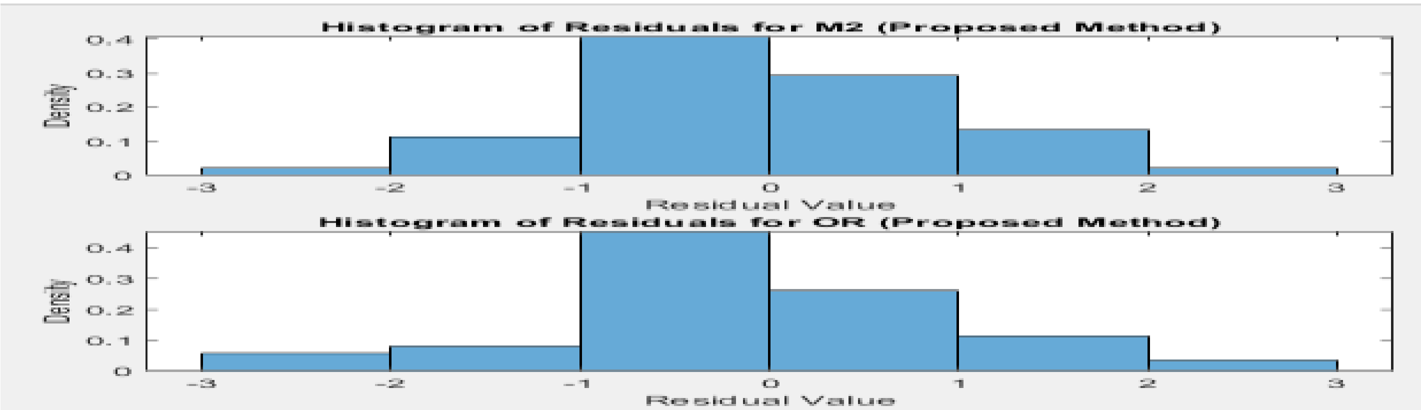

Figure 12: Histogram of the residuals for the Proposed VAR (2) Model for Dependent Variables M2 and OR.

The histogram in Figure 12 gives the residual distribution for the two dependent variables Broad Money (M2) and Official Reserves (OR) produced by the proposed VAR(1) model using the Coiflets1 wavelet.

Residuals of both variables cluster near zero, indicating a good model fit. The Jarque-Bera test confirms that the residuals follow a normal distribution, where the p-value for M2 is 0.5 and for OR is 0.20736, both of which are greater than 0.05 (P-Value > 0.05). These results show that conditions do not significantly deviate from normality since the model has been able to effectively capture all dynamic information without bias.

The diagnostic tests, which includes the residual sample autocorrelation function (ACF) and the histogram of residuals for the proposed VAR(1) model using the Coiflets1 wavelet, confirm its efficiency and accuracy in capturing the dynamic relationships within the data. These results show that the model achieves a normally distributed residual structure with minimal bias, enhancing its reliability and accuracy while confirming its suitability for future predictions.

Conclusions

From though the traditional and proposed VARMA models analyses using Discrete Wavelet Transform (DWT) are as follows:

1. The results demonstrated that wavelet analysis (Coiflets1) outperforms traditional methods in reducing the Mean Squared Error (MSE) and improving statistical information criteria (AIC and BIC), thereby enhancing estimation accuracy and noise reduction in approximate VAR models across all sample sizes (100, 500, and 1000), as indicated by the asterisk (*). These findings are based on simulation results derived from a normal distribution, obtained through 1000 simulation experiments for the bivariate and trivariate k-dimensional (k = 2,3) VAR(p) models with lag orders (p = 1, 2).

2. The study confirmed that wavelet analysis is effective in separating general trends from random fluctuations in time series data, leading to improved model stability and increased reliability in economic and financial forecasting.

3. The empirical results applied to Iraq's monthly economic data demonstrated that the VAR(1) model using Coiflets1 was the most efficient in balancing accuracy and computational complexity. Higher-order lag models, such as VAR(2) and VAR(3), did not yield significant improvements in accuracy, confirming that selecting a simpler model may be more effective when dealing with complex time-series data.

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