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Original Article

Improving Financial Volatility Modeling Using Neutrosophic Logic and Applying the GJR-GARCH Model

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ABSTRACT

This study adopts an innovative approach based on incorporating neutrosophic logic (NL) into the Glosten, Jagannathan, and Runkle generalized autoregressive conditional heteroskedasticity (GJR-GARCH) model to improve the estimation of conditional volatility in financial markets. It focuses on the analysis of time-varying financial volatility, considering account the effects of asymmetric shocks on variance. It suffers from difficulty in dealing with ambiguity and uncertainty in financial data. The incorporation of NL, which is based on three main elements: truth, error, and indeterminacy, allows for improving the accuracy of estimating unexpected and uncertain sharp volatility in markets. This new model provides a comprehensive framework that can better deal with the complexity and ambiguity of financial data. The results show that this methodology contributes to improves the model's ability to predict conditional volatility and provide evidence-based financial decisions, especially in complex financial environments. The suggested model in significant steps in enhancing the capacity to handle unforeseen financial shocks and gaining a greater knowledge of the complex behavior of financial data. As a result, it is a helpful tool to help decision-makers and investors make better, more accurate financial judgments.

Keywords: Financial Volatility, Neutrosophic Logic, Conditional Variance Models, Asymmetric Shocks, and Financial Forecasting.

1 Introduction

Conditional Heteroscedasticity variance models (shortly ARCH), which first introduced by [1], and were developed and generalized by [2] to analyze volatility in financial markets, which plays a fundamental role in economic forecasting and financial decision-making, as the ability to understand and predict conditional variance contributes to risk management and improved investment strategies. Among the statistical models that have proven effective in this field is the GJR-GARCH model, which was first introduced by [3], and deals with conditional volatility while taking into account the effects of asymmetric shocks on variance.

Deep, rooted ambiguity and uncertainty in financial data represent the main difficulties for the GJR-GARCH model. The prediction of volatility that occurs irregularly or caused by unexpected circumstances is regarded as one of these difficulties.

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We can solve these problems by using NL as an innovative mathematical framework we can analyze, ambiguous and uncertain data [4].

Truth, falsity, and indeterminacy are the main elements that NL-based on NL provides a flexible way to classify uncertain or contradictory information. Using this logic in the analysis of financial data, improvements in the understanding of conditional variance may be achieved by allowing for a more accurate representation of complexity and ambiguity [5-6].

The study tries to combine NL with the GJR-GARCH model to develop a much more flexible and accurate tool for analyzing conditional volatility in financial data. By using NL, this study aims to improve the estimation of uncertainty and ambiguity in financial data, which contributes to enhancing the predictive ability of the model. Its main goal is to present a model capable of handling data characterized by a high degree of uncertainty and unexpected fluctuations, thus presenting better and clearer financial decisions. Making in the market environment.

The first part included the introduction and the aims of the study. The second part included the neutrosophic regression model. In addition to a description of the data used in the study, including some tables and graphs that illustrate the results. The fourth and final part included the most important conclusions reached by the study.

2 Neutrosophic GJR-GARCH Model

In 1993, Glosten et al. proposed an improved version of the GARCH model and a special case of the asymmetric exponent model when $\delta = 2$ in the following Equation [5]:

$$y_t = \mu_t + x_t \quad (1)$$

$$x_t = \sigma_t \varepsilon_t$$

$$\sigma_t^\delta = w + \sum_{i=1}^{Q} \alpha_i \left(|x_{t-i}| - \lambda_i x_{t-i}\right)^\delta + \sum_{j=1}^{P} \beta_j \sigma_{t-j}^\delta \quad (2)$$

Equation 1 is called the mean equation, while equation (2) is called the instability equation (volatility), where y_t is the number of observations and μ_t is the conditional mean, σ_t is conditional variance, with positive constant w, and ε_t is white noises are normally distributed with zero expectation and constant variance. To capture the possible asymmetric effects of positive and negative surprises on the conditional variance. This means that there is one additional lever parameter, but this time the response to volatility is specifically rewritten in terms of only negative surprises. The model structure then becomes equation (2) by the form [7-8]:

$$\sigma_t = \sqrt{w + \sum_{i=1}^{Q} (\alpha_i + \lambda_i H_{x_{t-i}}) x_{t-i}^2 + \sum_{j=1}^{P} \beta_j \sigma_{t-j}^2} \quad (3)$$

Where α_i, β_j, and λ_i are the ARCH, GARCH, and Leverage effect parameters respectively, and $H_{x_{t-i}}$ is an indicator function has the form [9]:

$$H_{x_{t-i}} = \begin{cases} 1 \ if \ x_{t-i} < 0 \\ 0 \ if \ x_{t-i} \geq 0 \end{cases} \quad (4)$$

The model can be written as follows, which reflects the incorporation of the neutrosophic components into the GJR-GARCH model. In form:

$$\sigma_t^2 = (w_T, w_F, w_I) + \sum_{i=1}^{Q} \left((\alpha_{i_T}, \alpha_{i_F}, \alpha_{i_I}) + (\lambda_{i_T}, \lambda_{i_F}, \lambda_{i_I}) I_{x_{t-i}} \right) x_{t-i}^2 + \sum_{j=1}^{P} \left(\beta_{j_T}, \beta_{j_F}, \beta_{j_I} \right) \sigma_{t-j}^2 \quad (5)$$

Where

- $w_T, \alpha_{i_T}, \lambda_{i_T}$, and β_{j_T} represent the Truth neutrosophic components parameters.
- $w_F, \alpha_{i_F}, \lambda_{i_T}$, and β_{j_F} represent the Falsely neutrosophic components parameters.
- $w_I, \alpha_{i_I}, \lambda_{i_I}$, and β_{j_I} represent the Indeterminacy neutrosophic components parameters.

Using the neutrosophic parts, the classical GJR-GARCH model is extended to include the uncertainty of the conditional variance.

The stability condition for the GJR-GARCH neutrosophic model is:

$$\left[\sum_{i=1}^{Q} \left(\alpha_{i_T} + \frac{\lambda_{i_T}}{2}\right) + \sum_{j=1}^{P} \beta_{j_T}\right] < 1 \quad (6)$$

The Neutrosophic GJR-GARCH (NeGJR) model provides an advanced method for analyzing time-lapse data that includes uncertainty and ambiguity. By incorporating the neutral components, the model can:

The advanced method of analyzing data that includes uncertainty values and ambiguity values of the GJR-GARCH Neutrosophic model can as well as neutral components to the model, which in work turn improves forecast accuracy in uncertain scenarios by allowing uncertainty and error levels to be calculated rather than providing only categorical estimates. It also provides a more adaptable model to deal with fluctuations in data series, which can enhance volatility forecasts. A deeper understanding of ambiguity improves the ability to evaluate data that consists ambiguous or partial information, allowing for a more comprehensive examination of trends and patterns.

3 Algorithm Application for Neutrosophic GJR-GARCH

This model can be applied by following the following steps to find a logical and appropriate analysis of the proposed model using the following steps:

1. Set a data set
2. To make a stability for the data set converting to returns series by form:

   $$r_t = \log u_t - \log u_{t-1} \quad (7)$$

   Where u_{t-1}, and u_t are observations at time $t-1$, and t respectively.
3. Calculating the neutrosophic parts by putting T as the Truth part, F for the Error part, and I for Indeterminacy part by respectively forms:

   $$T = r_t \quad (8)$$

   $$F = 1 - T \quad (9)$$

   $$I = \frac{|T - F|}{2} \quad (10)$$
4. Estimate the neutrosophic parameters of the GJR-GARCH model
5. Calculate the conditional variance: After applying the model, the conditional variance is calculated using the modified equation for the true, false and uncertainty values as follows:

$$\sigma_T^2 = \frac{w_T}{1 - \sum_{i=1}^{q}\left(\alpha_{i_T} + \frac{\gamma_{i_T}}{2}\right) + \sum_{j=1}^{p} \beta_j} \quad (11)$$

This variance should give a more accurate picture of volatility under conditions that include uncertainty and ambiguity.

6. Finally, use the neutrosophic model- GJR-GARCH to forecast the future conditional variance based on the available data.

7. The original data are plotted with the neutrosophic values, as well as the inferred and predicted conditional variance.

4 Application

On a practical level, and to know the efficiency of the proposed model, a practical aspect that was carried out by modeling that simulates the structure of the neutrosophic model-GJR-GARCH model. The practical application includes the steps of the model's algorithm, along with some tests during the analysis stages.

The data used in the study represented the real data for the monthly average of pounds sterling (in millions) spent by foreign visitors in the United Kingdom for the period from January 1986 to February 2020, with 407 views. Where figure 1 show the time series Data.

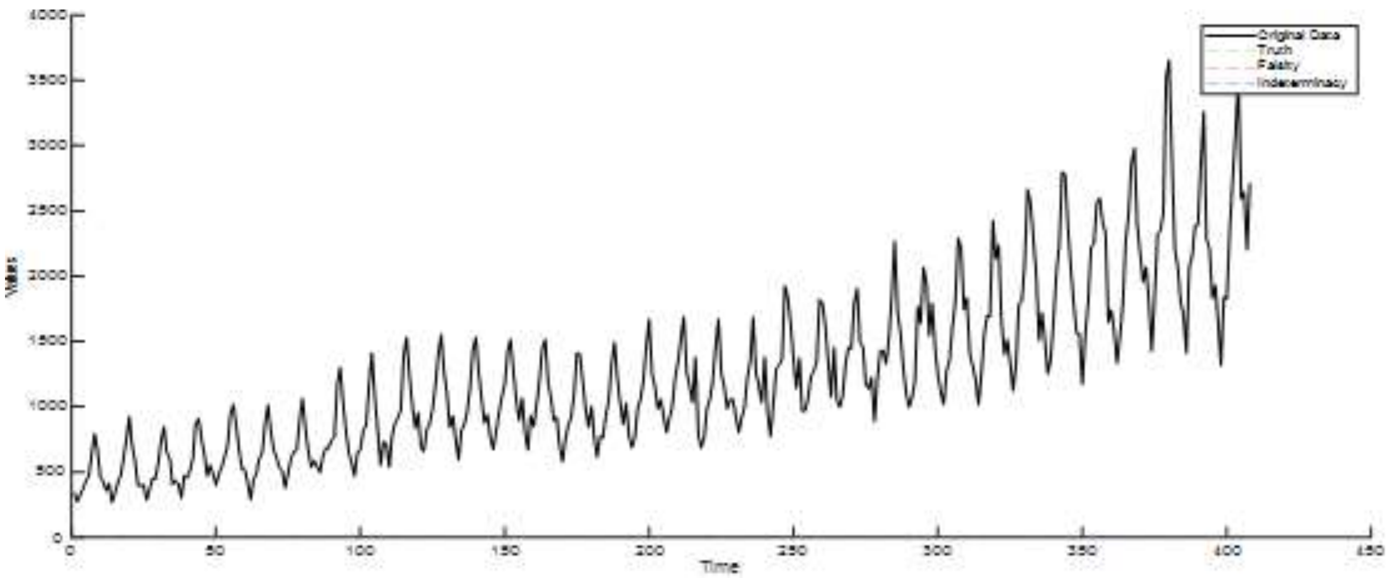

Figure 1: The time series Data.

This figure represents monthly data on average expenditure in pounds sterling by overseas visitors to the UK from January 1986 to February 2020. The graph shows clear fluctuations over time, with periods of sharp increases and decreases, reflecting fluctuations in the economy and tourism. This type of data is commonly used to model volatility.

The second step involves plotting the total autocorrelation function (ACF) for twenty time lags to fine-tune the nature of the data. Figure 2 represents the ACF plot for the data.

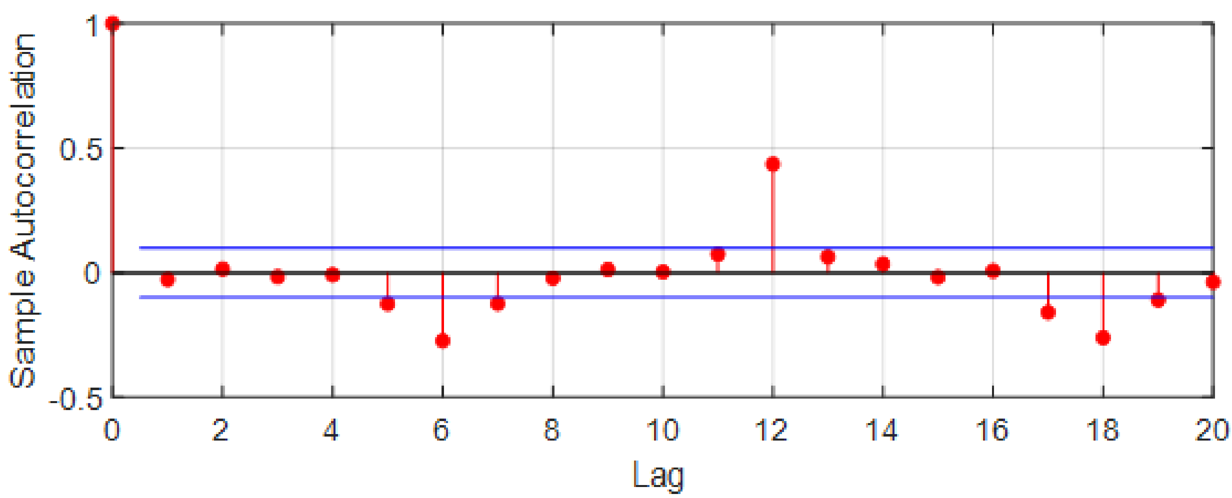

Figure 2: ACF plot for the data.

The ACF plot shows the temporal correlations between different values in the time series across 20 lags. The results suggest that the original data is non-stationary because many of the lags fall outside the confidence limits [10]. This means that there is a strong temporal correlation between the points, which requires data processing to convert them to stationary data.

Now the data series is converted to a return series as in equation (7), then the true values are found as in equation (8), false as in equation (9), and the Indeterminacy values are found as in equation (10). Tables 1, 2, and 3 represent the values of the True, False, and Indeterminacy table.

Table 1: Truth

↦									
-0.2298	0.1817	0.1351	0.1518	0.0884	0.3102	0.2066	0.1881-	-0.3543	-0.0790
-0.1767	0.1433	-0.4375	0.2366	0.2082	0.1371	0.2415	0.2065	0.2164	-0.2787
-0.1795	-0.3877	-0.0051	0.0151	-0.3450	0.2427	0.2198	-0.0134	0.2423	0.2595
0.1406	-0.2651	-0.0686	-0.4055	0.0739	-0.0592	-0.3024	0.4390	0.0302-	0.1040
0.1787	0.3667	0.0404	-0.1856	-0.1731	-0.3030	0.1666	-0.1310	-0.1998	0.1915
0.1062	0.1391	0.1307	0.2919	0.0797	-0.1710	-0.2473	-0.2420	-0.0192	-0.2106
-0.3691	0.4181	0.1140	0.2051	0.0672	0.2919	0.1579	-0.2212	-0.1997	-0.0847
-0.1385	-0.0481	-0.2963	0.2743	0.1970	0.0841	0.0151	0.3085	0.1576	-0.2438
-0.2389	-0.2020	0.0926	-0.0778	-0.0924	0.2199	0.0985	0	0.0884	0.0278
0.4253	0.1037	-0.2193	-0.2464	-0.2244	-0.1362	-0.1954	0.3300	0.0258	0.2014
0.0490	0.3067	0.1921	-0.2781	-0.2637	-0.3982	0.2809	-0.0221	-0.2881	0.3739
0.1151	0.0502	0.0581	0.3600	0.0923	-0.2149	-0.1991	-0.1911	0.1331	-0.3250
-0.0490	0.2313	0.0483	0.1294	0.1621	0.1876	0.1009	-0.1997	-0.1454	-0.2699
0.1040	-0.2109	-0.2402	0.3217	0.0547	0.1064	0.1830	0.2186	0.0647	-0.1947
-0.1793	-0.1873	0.0749	-0.2029	-0.1333	0.1828	0.1671	0.1184	0.0847	0.1965
0.0626	-0.1755	-0.1937	-0.1589	0.1769	-0.2706	-0.1977	0.3320	-0.0934	0.1738
0.1792	0.1800	0.0480	-0.2446	-0.1094	-0.1752	0.0265	-0.2758	-0.1858	0.2618
0.1367	0.0641	0.1863	0.2435	-0.0085	-0.1601	-0.1979	-0.1474	0.1726	-0.2748
-0.2127	0.2270	-0.0130	0.2011	0.1172	0.2381	0.1169	-0.2089	-0.1692	-0.1686
0.1715	-0.2710	-0.1335	0.1011	0.2586	0.0726	0.1202	0.2069	0.1315	-0.2893
-0.0789	-0.1630	0.0716	-0.1303	-0.1424	0.1260	0.0881	0.2045	0.0952	0.1263
0.1021	-0.2930	-0.0861	-0.1135	0.2906	-0.5654	-0.1335	0.1011	0.2586	0.0726
0.1202	0.2069	0.1315	-0.2893	-0.0789	-0.1630	0.0716	0	-0.1303	-0.1424
0.1260	0.0881	0.2045	0.0952	0.2285	-0.2930	-0.0861	-0.1135	0.2906	-0.3639
-0.2169	0.2844	0.2297	0.0177	0.0381	0.3438	-0.0457	-0.1164	-0.1876	-0.1742
0.1882	-0.3496	0.0010	0.0868	0.1570	0.0216	0.0672	0.2958	-0.0144	-0.1020
-0.1920	-0.2172	0.3004	-0.3258	-0.0518	0.0856	0.2051	0.0869	-0.0041	0.2161
0.0594	-0.2442	-0.0237	-0.2175	-0.0303	0.0745	-0.3214	0.3140	0.1599	0
-0.0718	0.1316	0.2029	0.1995	-0.2767	-0.0926	-0.1953	-0.1531	-0.1058	0.0717
0.1039	0.3987	-0.0752	0.2315	-0.0573	-0.2353	0.1508	-0.2144	-0.1552	-0.1136
-0.0839	0.2300	0.0510	0.1821	0.1062	0.2429	-0.0333	-0.2401	0.0514	-0.2670
-0.0616	-0.0869	-0.1756	0.2017	0.2201	0.0901	-0.0024	0.3609	-0.1273	0.0466
-0.2976	-0.1715	0.0783	-0.1251	-0.1737	0.1684	0.2950	0.0111	0.1466	0.2434
-0.0378	-0.1137	-0.1279	-0.2910	0.1334	-0.1340	-0.1831	0.0819	0.2016	0.1992
0.0957	0.2239	-0.0068	-0.1538	-0.1457	-0.1383	-0.1379	-0.0051	-0.2838	0.2793
0.1520	0.2096	0.0130	0.1302	0.0136	-0.0743	-0.0329	-0.3486	0.0555	-0.0932
-0.1745	0.1379	0.1559	0.2392	0.0869	0.1456	0.0400	-0.2210	-0.0938	-0.1066
0.0552	-0.1273	-0.2440	0.2489	0.2400	0.0090	0.0564	0.3497	0.0371	-0.2266
-0.2789	-0.0598	-0.1427	-0.0525	-0.1903	0.3845	0.0392	0.1005	0.0046	0.1784
0.1292	-0.3581	-0.0294	-0.1896	0.0537	-0.1597	-0.2222	0.3337	-0.0087	0.2521
0.1493	0.1257	0.1274	-0.3028	0.0199	-0.1838	0.2073			

Table 2: Falsity

↦									
1.2298	0.8183	0.8649	0.8482	0.9116	0.6898	0.7934	1.1881	1.3543	1.0790
1.1767	0.8567	1.4375	0.7634	0.7918	0.8629	0.7585	0.7935	0.7836	1.2787
1.1795	1.3877	1.0051	0.9849	1.3450	0.7573	0.7802	1.0134	0.7577	0.7405
0.8594	1.2651	1.0686	1.4055	0.9261	1.0592	1.3024	0.5610	1.0302	0.8960
0.8213	0.6333	0.9596	1.1856	1.1731	1.3030	0.8334	1.1310	1.1998	0.8085
0.8938	0.8609	0.8693	0.7081	0.9203	1.1710	1.2473	1.2420	1.0192	1.2106
1.3691	0.5819	0.8860	0.7949	0.9328	0.7081	0.8421	1.2212	1.1997	1.0847
1.1385	1.0481	1.2963	0.7257	0.8030	0.9159	0.9849	0.6915	0.8424	1.2438
1.2389	1.2020	0.9074	1.0778	1.0924	0.7801	0.9015	1.0000	0.9116	0.9722
0.5747	0.8963	1.2193	1.2464	1.2244	1.1362	1.1954	0.6700	0.9742	0.7986

0.9510	0.6933	0.8079	1.2781	1.2637	1.3982	0.7191	1.0221	1.2881	0.6261
0.8849	0.9498	0.9419	0.6400	0.9077	1.2149	1.1991	1.1911	0.8669	1.3250
1.0490	0.7687	0.9517	0.8706	0.8379	0.8124	0.8991	1.1997	1.1454	1.2699
0.8960	1.2109	1.2402	0.6783	0.9453	0.8936	0.8170	0.7814	0.9353	1.1947
1.1793	1.1873	0.9251	1.2029	1.1333	0.8172	0.8329	0.8816	0.9153	0.8035
0.9374	1.1755	1.1937	1.1589	0.8231	1.2706	1.1977	0.6680	1.0934	0.8262
0.8208	0.8200	0.9520	1.2446	1.1094	1.1752	0.9735	1.2758	1.1858	0.7382
0.8633	0.9359	0.8137	0.7565	1.0085	1.1601	1.1979	1.1474	0.8274	1.2748
1.2127	0.7730	1.0130	0.7989	0.8828	0.7619	0.8831	1.2089	1.1692	1.1686
0.8285	1.2710	1.1335	0.8989	0.7414	0.9274	0.8798	0.7931	0.8685	1.2893
1.0789	1.1630	0.9284	1.1303	1.1424	0.8740	0.9119	0.7955	0.9048	0.8737
0.8979	1.2930	1.0861	1.1135	0.7094	1.5654	1.1335	0.8989	0.7414	0.9274
0.8798	0.7931	0.8685	1.2893	1.0789	1.1630	0.9284	1.0000	1.1303	1.1424
0.8740	0.9119	0.7955	0.9048	0.7715	1.2930	1.0861	1.1135	0.7094	1.3639
0.7156	0.7703	0.9823	0.9619	0.6562	1.0457	1.1164	1.1876	1.1742	0.8118
1.3496	0.9990	0.9132	0.8430	0.9784	0.9328	0.7042	1.0144	1.1020	1.1920
1.2172	0.6996	1.3258	1.0518	0.9144	0.7949	0.9131	1.0041	0.7839	0.9406
1.2442	1.0237	1.2175	1.0303	0.9255	1.3214	0.6860	0.8401	1.0000	1.0718
0.8684	0.7971	0.8005	1.2767	1.0926	1.1953	1.1531	1.1058	0.9283	0.8961
0.6013	1.0752	0.7685	1.0573	1.2353	0.8492	1.2144	1.1552	1.1136	1.0839
0.7700	0.9490	0.8179	0.8938	0.7571	1.0333	1.2401	0.9486	1.2670	1.0616
1.0869	1.1756	0.7983	0.7799	0.9099	1.0024	0.6391	1.1273	0.9534	1.2976
1.1715	0.9217	1.1251	1.1737	0.8316	0.7050	0.9889	0.8534	0.7566	1.0378
1.1137	1.1279	1.2910	0.8666	1.1340	1.1831	0.9181	0.7984	0.8008	0.9043
0.7761	1.0068	1.1538	1.1457	1.1383	1.1379	1.0051	1.2838	0.7207	0.8480
0.7904	0.9870	0.8698	0.9864	1.0743	1.0329	1.3486	0.9445	1.0932	1.1745
0.8621	0.8441	0.7608	0.9131	0.8544	0.9600	1.2210	1.0938	1.1066	0.9448
1.1273	1.2440	0.7511	0.7600	0.9910	0.9436	0.6503	0.9629	1.2266	1.2789
1.0598	1.1427	1.0525	1.1903	0.6155	0.9608	0.8995	0.9954	0.8216	0.8708
1.3581	1.0294	1.1896	0.9463	1.1597	1.2222	0.6663	1.0087	0.7479	0.8507
0.7298	0.8743	0.8726	1.3028	0.9801	1.1838	0.7927			

Table 3: Indeterminacy.

$\mapsto$									
0.7298	0.3183	0.3649	0.3482	0.4116	0.1898	0.2934	0.6881	0.8543	0.5790
0.6767	0.3567	0.9375	0.2634	0.2918	0.3629	0.2585	0.2935	0.2836	0.7787
0.6795	0.8877	0.5051	0.4849	0.8450	0.2573	0.2802	0.5134	0.2577	0.2405
0.3594	0.7651	0.5686	0.9055	0.4261	0.5592	0.8024	0.0610	0.5302	0.3960
0.3213	0.1333	0.4596	0.6856	0.6731	0.8030	0.3334	0.6310	0.6998	0.3085
0.3938	0.3609	0.3693	0.2081	0.4203	0.6710	0.7473	0.7420	0.5192	0.7106
0.8691	0.0819	0.3860	0.2949	0.4328	0.2081	0.3421	0.7212	0.6997	0.5847
0.6385	0.5481	0.7963	0.2257	0.3030	0.4159	0.4849	0.1915	0.3424	0.7438
0.7389	0.7020	0.4074	0.5778	0.5924	0.2801	0.4015	0.5000	0.4116	0.4722
0.0747	0.3963	0.7193	0.7464	0.7244	0.6362	0.6954	0.1700	0.4742	0.2986
0.4510	0.1933	0.3079	0.7781	0.7637	0.8982	0.2191	0.5221	0.7881	0.1261
0.3849	0.4498	0.4419	0.1400	0.4077	0.7149	0.6991	0.6911	0.3669	0.8250
0.5490	0.2687	0.4517	0.3706	0.3379	0.3124	0.3991	0.6997	0.6454	0.7699
0.3960	0.7109	0.7402	0.1783	0.4453	0.3936	0.3170	0.2814	0.4353	0.6947
0.6793	0.6873	0.4251	0.7029	0.6333	0.3172	0.3329	0.3816	0.4153	0.3035
0.4374	0.6755	0.6937	0.6589	0.3231	0.7706	0.6977	0.1680	0.5934	0.3262
0.3208	0.3200	0.4520	0.7446	0.6094	0.6752	0.4735	0.7758	0.6858	0.2382
0.3633	0.4359	0.3137	0.2565	0.5085	0.6601	0.6979	0.6474	0.3274	0.7748
0.7127	0.2730	0.5130	0.2989	0.3828	0.2619	0.3831	0.7089	0.6692	0.6686
0.3285	0.7710	0.6335	0.3989	0.2414	0.4274	0.3798	0.2931	0.3685	0.7893
0.5789	0.6630	0.4284	0.6303	0.6424	0.3740	0.4119	0.2955	0.4048	0.3737
0.3979	0.7930	0.5861	0.6135	0.2094	1.0654	0.6335	0.3989	0.2414	0.4274
0.3798	0.2931	0.3685	0.7893	0.5789	0.6630	0.4284	0.5000	0.6303	0.6424
0.3740	0.4119	0.2955	0.4048	0.2715	0.7930	0.5861	0.6135	0.2094	0.8639
0.7169	0.2156	0.2703	0.4823	0.4619	0.1562	0.5457	0.6164	0.6876	0.6742
0.3118	0.8496	0.4990	0.4132	0.3430	0.4784	0.4328	0.2042	0.5144	0.6020
0.6920	0.7172	0.1996	0.8258	0.5518	0.4144	0.2949	0.4131	0.5041	0.2839
0.4406	0.7442	0.5237	0.7175	0.5303	0.4255	0.8214	0.1860	0.3401	0.5000
0.5718	0.3684	0.2971	0.3005	0.7767	0.5926	0.6953	0.6531	0.6058	0.4283
0.3961	0.1013	0.5752	0.2685	0.5573	0.7353	0.3492	0.7144	0.6552	0.6136
0.5839	0.2700	0.4490	0.3179	0.3938	0.2571	0.5333	0.7401	0.4486	0.7670
0.5616	0.5869	0.6756	0.2983	0.2799	0.4099	0.5024	0.1391	0.6273	0.4534

0.7976	0.6715	0.4217	0.6251	0.6737	0.3316	0.2050	0.4889	0.3534	0.2566
0.5378	0.6137	0.6279	0.7910	0.3666	0.6340	0.6831	0.4181	0.2984	0.3008
0.4043	0.2761	0.5068	0.6538	0.6457	0.6383	0.6379	0.5051	0.7838	0.2207
0.3480	0.2904	0.4870	0.3698	0.4864	0.5743	0.5329	0.8486	0.4445	0.5932
0.6745	0.3621	0.3441	0.2608	0.4131	0.3544	0.4600	0.7210	0.5938	0.6066
0.4448	0.6273	0.7440	0.2511	0.2600	0.4910	0.4436	0.1503	0.4629	0.7266
0.7789	0.5598	0.6427	0.5525	0.6903	0.1155	0.4608	0.3995	0.4954	0.3216
0.3708	0.8581	0.5294	0.6896	0.4463	0.6597	0.7222	0.1663	0.5087	0.2479
0.3507	0.3743	0.3726	0.8028	0.4801	0.6838	0.2927			

Table 1 shows the calculated values of "truth" in the financial returns series. Positive values indicate periods of market buoyancy, where returns are positive. Negative values indicate periods of decline in returns. This table helps to identify periods when markets experienced large fluctuations in returns.

Table 2 shows the inverse or "error" values calculated from the returns series. These values are calculated as the inverse of the truth values (1 - Truth). High values of "error" indicate periods when the model's predictions were inaccurate, meaning that the markets did not match the predictions or calculated results.

Table 3 shows the calculated values of ambiguity (uncertainty) in the data. These high ambiguity values indicate that there is an overlap between the truth and error components, reflecting the uncertainty in the predictions during those periods. Low values indicate that the data were more clear and stable.

Below is a graphic of the values of the True, False, and Indeterminacy.

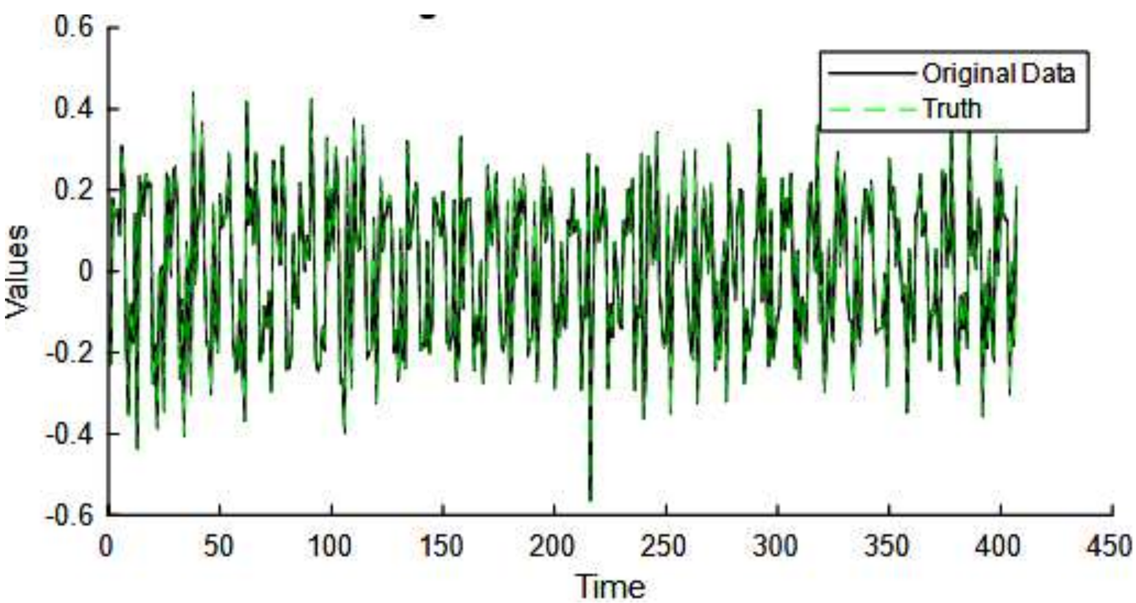

Figure 3: True with the return series.

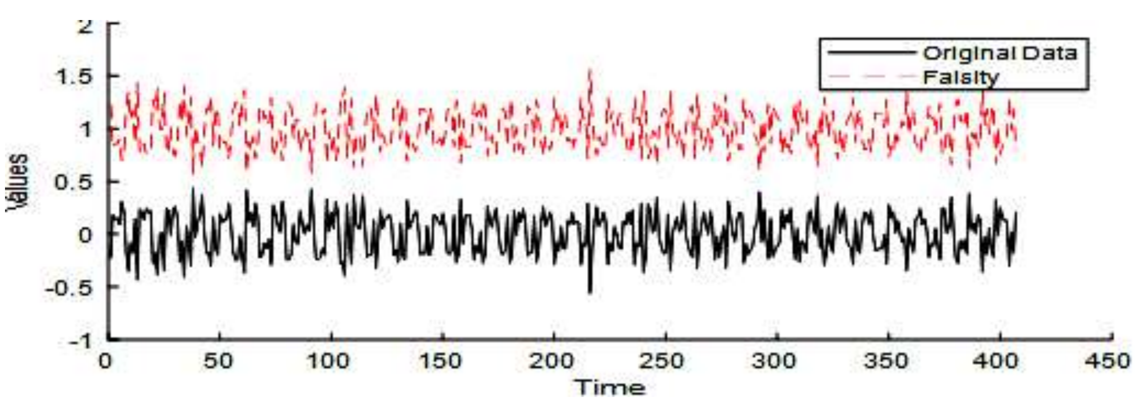

Figure 4: False with return series.

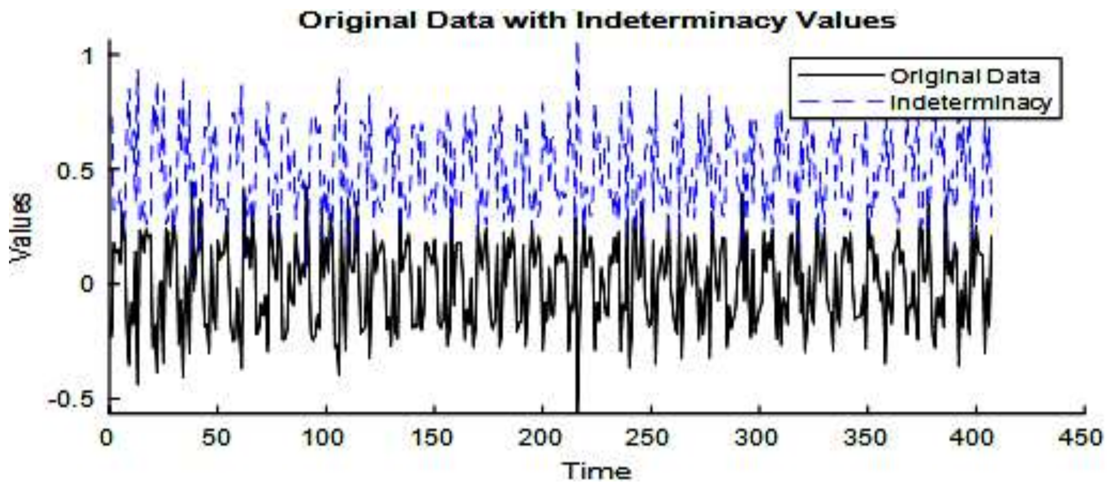

Figure 5: Indeterminacy with return series.

Shows the values that represent the “true” or actual values of the returns calculated from the original data. Positive values indicate periods of financial market booms, while negative values indicate periods of market downturns. This gives a clear picture of the true changes that occur in the data over time.

Figure 4 shows the inverse values of the returns or so-called “error” component. These values are calculated as 1 - Truth. They indicate periods when the model was not accurate in its prediction, indicating times when financial forecasts were not consistent with actual results. The inverse values show a different behavior compared to the true values, which helps to evaluate the performance of the model.

Figure 5 shows the uncertainty (ambiguity) between the truth and error values, and shows how much overlap there is between these two components. It shows periods when forecasts are unclear or ambiguous. High ambiguity values indicate that the market in those periods was experiencing uncertainty or unpredictability.

Now a visual test is performed to check whether the data series has gained stability or not by plotting the ACF function for the return series, where Figure 6 represents the plot of the ACF function for the return series.

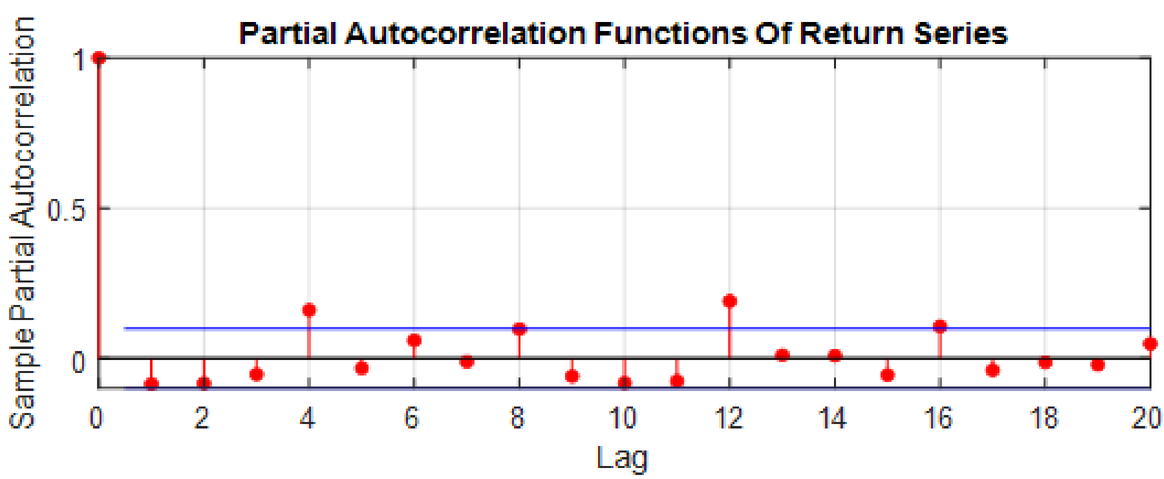

Figure 6: the ACF function for the return series.

After transforming the original data into a return series, this plot shows the ACF of the returns across 20 offset periods. It shows that the data is stationary after the transformation, as many of the values are now within the confidence limits, meaning that the model is suitable for volatility analysis.

Now using Akaike criteria (AIC) and Bayesian Information Criterion (BIC) [10-12] to find out the best model, the following table represents the criteria for the best rank of the model.

Table 4: AIC and BIC criteria for NeGJR(q,p).

NeGJR(q,p)	AIC			BIC		
	Troth	Falsity	Indeterminacy	Troth	Falsity	Indeterminacy
NeGJR(1,1)	-173.0103	1173.4644	646.5677	-156.975	1189.4996	662.6029
NeGJR(1,2)	-169.0103	1177.4343	650.4247	-144.9574	1201.4872	674.4776
NeGJR(1,3)	-166.7905	1181.4343	654.4247	-134.72	1213.5048	686.4952
NeGJR(1,4)	-167.7096	1185.4343	658.4247	-127.6214	1225.5224	698.5128
NeGJR(2,1)	-171.0103	1175.4644	648.5677	-150.9662	1195.5084	668.6117
NeGJR(2,2)	-167.1103	1179.4343	652.4247	-138.9486	1207.496	680.4864
NeGJR(2,3)	-164.7905	1183.4343	656.4247	-128.7112	1219.5136	692.504
NeGJR(3,1)	-170.5704	1177.4644	650.5677	-146.5175	1201.5173	674.6205
NeGJR(3,2)	-166.8471	1181.4343	654.4247	-134.7766	1213.5048	686.4952
NeGJR(3,3)	-163.0243	1185.4343	658.4247	-122.9362	1225.5224	698.5128
NeGJR(4,1)	-171.0003	1179.4644	652.5677	-143.5024	1207.5261	680.6293
NeGJR(4,2)	-167.5641	1183.4343	656.4247	-131.4847	1219.5136	692.504
NeGJR(4,3)	-163.5641	1187.4343	660.4247	-119.4671	1231.5312	704.5216

Table 4 shows a comparison of different NeGJR models using Akaike Information Criteria (AIC) and Bayesian Information Criterion (BIC) to determine the best model. The NeGJR(1,1) model is considered the best based on the lowest values of AIC and BIC. This means that this model provides the best fit to the data and is therefore the most efficient in predicting financial volatility. Accordingly, the estimated parameters and some tests for this model are:

Table 5: The value of parameters, with some tests.

parameter	Value	Standard-Error	T-Statistic	P-Value
w_T	0.036785	0.1115	0.32991	0.74147
α_{1_T}	2e-12	2.9763	6.7198e-13	1
β_{1_T}	2e-12	0.10424	1.9186e-11	1
λ_{1_T}	0.040828	0.1235	0.3306	0.74095
w_F	0.84279	5.2792	0.15964	0.87316
α_{1_F}	0.047686	5.2137	0.0091462	0.9927
β_{1_F}	0.13088	0.7457	0.17551	0.86068
λ_{1_F}	0.87166	165	0.0052827	0.99579
w_I	0.19959	0.34968	0.57076	0.56816
α_{1_I}	0.14673	1.2668	0.11583	0.90778
β_{1_I}	0.146	0.2311	0.63175	0.52755
λ_{1_I}	1	20.887	0.047877	0.96181

Table 5 contains the estimated parameters of the NeGJR(1,1) model, including (w, α, β, λ) with the associated statistical values (T-statistic, P-value). The table reflects the accuracy of the model and the ability to interpret financial data. The calculated values show that the model meets the stability condition, which means that the results obtained from the model can be trusted. The low values of T-statistic and P-value indicate that some parameters are not statistically significant, which means that there may be room for improvement in the model

Now the model stability condition is applied and the unconditional variance value of the model is calculated according to equations (6) and (11) as follows:

$$\alpha_{1_T} + \beta_{1_T} + \frac{\lambda_{1_T}}{2} = 2\mathrm{e} - 12 + 2\mathrm{e} - 12 + 0.040828 = 0.0408 < 1$$

$$\sigma^2 = \frac{w}{1-\left(\alpha_{1_T}+\beta_{1_T}+\frac{\lambda_{1T}}{2}\right)} = \frac{0.036785}{1-0.0408} = 0.0383496664 \approx 0.038$$

From the above equation notice the model is stable, i.e. the condition in equation (6) is hold, and the unconditional variance value of the Neutrosophic GJR-GARCH hold equation (11).

After verifying the stability conditions of the model, the next step is to infer the values of Troth, Falsity, and indeterminacy with unconditional variance for the stable and best model, as in the following figures:

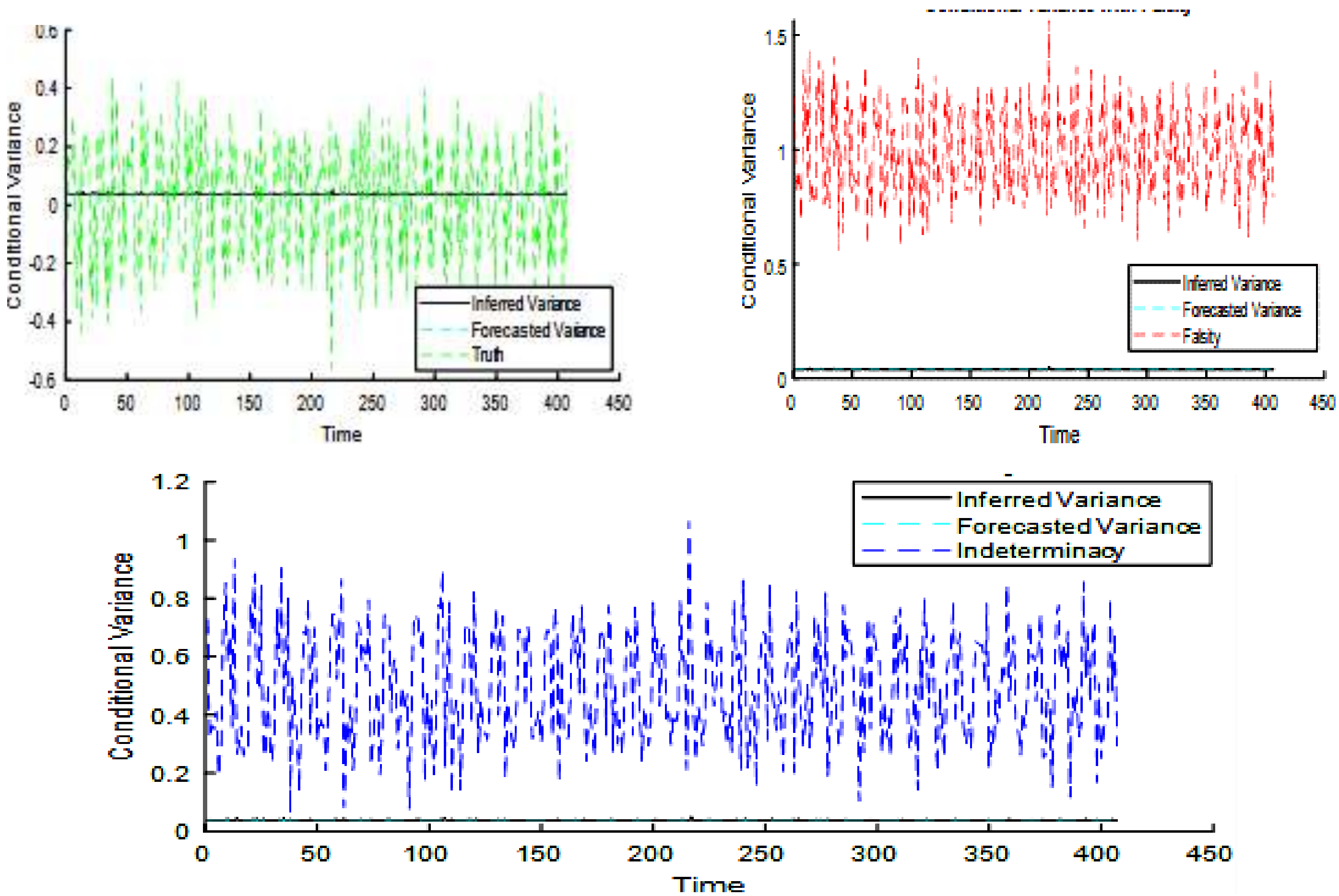

Figure 7: The values of Troth, Falsity, and indeterminacy with unconditional variance for the stable model.

This figure shows the three neutrosophic values (truth, error, and ambiguity) along with the unconditional variance of the model. It shows how these components interact together to give a more comprehensive representation of financial volatility, with the unconditional variance indicating the stability of the model.

Now the unconditional variance of the original series is predicted by reversing equations (7), (8) and (9) to obtain the predicted value of the conditional variance of the stationary model, as shown in the following figure.

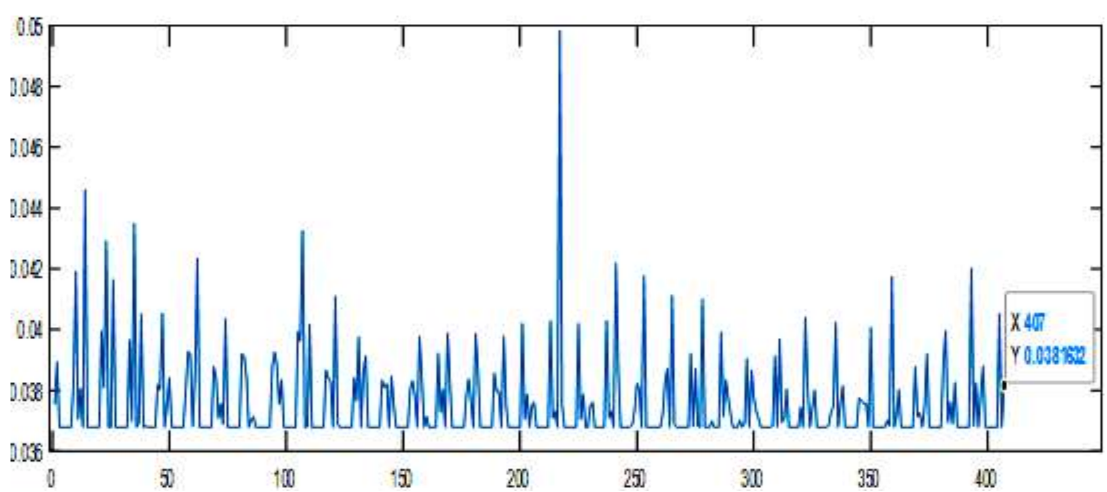

Figure 8: The unconditional variance for the stable model.

The figure shows the unconditional variance forecast of the stable model using values calculated from the NeGJR model. The stable model is us to forecast future financial volatility, as shown in the above figure, which gives an accurate imageof the variance expectations for the coming period based on the results.

5 Conclusions

The results of the paper explain that the accuracy of producing conditional variance in financial data increases when NL is combined with a GJR-GARCH model. The improved model provides a more comprehensive depiction of unexpected market surprises and financial fluctuations. The understanding of the uncertain behavior for financial data is enhanced by the use of NL, which addresses Truthiness, Falseness, and uncertainty. This method improves the ability of models to respond to unexpected shocks, and Leverage effects by better handling the ambiguity prepared in financial data. The study also shows that the improved model meets the stability requirements, indicating that it can be relied upon to predict the future and estimate conditional variance. Unconditional variance was used to verify the stability of the models, and the results were reliable and consistent. The proposed model can be widely applied to financial market research. The NeGJR(1,1) model is the most accurate and suitable model for evaluating complex financial data, improving the ability to predict changes in financial markets, and enhancing financial decision-making, as defined by the AIC and BIC criteria.

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