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Original Article

A New Formula for Friction Factor-Reynolds Number of Non-Newtonian Fluids

Ahmed A. Ouda [1], Ehab S. Hussein [1], Ahmed J. Shkarah * [1], Mustafa M. Mansour [1]

[1] *Department of Mechanical Engineering, College of Engineering, University of Thi-Qar, Thi-Qar, 64001, Iraq.*

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ABSTRACT

This study presents a new unified correlation to describe the relationship between the friction factor and Reynolds number for non-Newtonian fluids across laminar, transitional, and turbulent flow regimes. The proposed formula is developed using power-law-based curve fitting, integrating multiple regimes into a continuous function with coefficients dependent on the flow behavior index (n). Unlike existing models such as Metzner-Otto and Joseph-Yang, this approach improves prediction accuracy for a wide range of non-Newtonian fluids, including shear-thinning and shear-thickening types. The model was validated against experimental data, demonstrating a maximum relative error of 7.15% and improved accuracy particularly for low-index (n < 1) fluids. This correlation enables accurate flow regime classification and simplifies friction factor estimation without iterative methods, offering practical advantages in fluid transport system design.

Keywords: Power law index, Flow regime classification, Generalized Reynolds number, rheology

1 Introduction

Understanding the relationship between the friction factor and Reynolds number is crucial for modeling fluid transport in engineering systems. While this relationship is well-established for Newtonian fluids, non-Newtonian fluids such as slurries, polymer solutions, and biological fluids exhibit complex flow behavior due to variable viscosity, making conventional models inadequate.

Several correlations have been proposed to extend friction factor-Reynolds number relations to non-Newtonian flows. The Metzner and Otto method introduced a generalized Reynolds number, but it provides limited accuracy for fluids with low flow behavior indices (n < 1). Similarly, models like Joseph and Yang's logistic dose approach work well for Newtonian fluids but lack flexibility across the full spectrum of non-Newtonian behaviors. These existing correlations often require regime-specific equations, piecewise definitions, or iterative solutions — complicating engineering design and simulation processes.

One method to estimate the importance of inertial and viscous forces in a specific flow scenario is by using the dimensionless Reynolds number (Re), which is a ratio of the two [1]. Although the concept was initially put forth by George Gabriel Stokes in 1851, it was Osborne Reynolds (1842-1912) who brought the Reynolds number into widespread use in 1883 and is so named after him [2]. There are many formulas for Reynolds number difference as well as models difference, but all have the same basic definition where: [3]

$$Re = \frac{inertial\ forces}{viscous\ forces} \quad (1)$$

Osborne Reynolds formula is

$$Re = \frac{\rho v L}{\mu} = \frac{vL}{\upsilon} \quad (2)$$

But this formula used for Newtonian fluids only [4]. Nikuradse (1950) at this research looked at the turbulent flow of water in pipes of different sizes, Reynolds numbers, and roughness levels. It described the length of a mixture using Prandtl's formula and showed the laws of resistance and velocity. Using Reynolds numbers, the outcomes were examined. [5]

Metzner (1957) In order to examine data on power consumption for three non-Newtonian fluids, the study set out to determine a correlation between fluid shear rate and impeller speed. Fast mixing of non-Newtonian fluids required far more power than Newtonian materials, according to the data, demonstrating that fluid shear rate was the most important parameter in predicting material behavior. [8]

Garcia (1986) In order to estimate the pressure drop in the pipe flow of non-Newtonian fluids, this article explained why friction variables were so important. It showed that flow behavior index, Reynolds number, and Hedstrom number impacted the predicted

* Corresponding author
E-mail address shkarah@utq.edu.iq (Instructor).
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friction factor in a considerable way. Using relationships without yield stress could also lead to errors, as this demonstrated. [10]

Madlener (2009) an expanded form of the generalized Reynolds number was used to characterize the Herschel-Bulkley-expanded (HBE) class of non-Newtonian gelled fluids. This parameter was used to compute the onset of turbulent flow in Newtonian and non-Newtonian fluids of the Herschel-Bulkley, Ostwald-de-Waele, and Bingham types, as well as in continuous flows. [12]

Burger (2010) this research examined three non-Newtonian fluids flowing laminarly in an open channel and how their cross-sectional shapes affected the friction factor–Reynolds number relationship. The findings aligned with earlier research on open channel flow of Newtonian fluids and demonstrated that the K values for triangular, semicircular, rectangular, and trapezoidal channels were 14.6, 16.2, 16.4, and 17.6, respectively.[17]

Joseph (2010) an exact formula for the association of composite friction factor with Reynolds number for laminar, transitional, and turbulent flow in smooth pipes was presented in this study. The formula was produced using a logistic dose curve technique that connected linear splines in log-log coordinates. All available smooth pipe flow data showed that the correlation was consistently applicable and as accurate, if not more accurate, than previous correlations within the range of Reynolds numbers. Rational fractions of power laws were the result of the expanded fitting approach.[16]

Dosunmu (2013) when calculating the amount of hydraulic horsepower needed and the pressure at the wellhead, friction pressure calculation was an essential component of pipe and annular hydraulics. Non-Newtonian fluids in laminar flow were described using an equation of the Hagen-Poiseuille type. The Dodge-Metzner equation was one example of a correlation that had been created for smooth pipes. Ignoring the effect of pipe roughness on wellbore hydraulics, earlier research had concentrated on non-Newtonian fluid friction factor equations for pipes that were smooth. Analyzing equivalent diameter definitions for circular flow, this study investigated friction factor correlations with relative roughness.[19]

Assefa (2014) in this study, the correlations of friction factors in smooth pipes throughout all Bingham fluid flow regimes were examined. The experimental findings were compared with nine selected models. For Reynolds numbers up to 40,000, the models proposed by Wilson-Thomas (1985) and Morrison (2013) performed best. The Wilson-Thomas model exhibited larger disparities outside of this range. When estimating the friction factor in smooth pipes, the Morrison model was preferred over the Moody chart and other implicit formulas, as it used fewer parameters and required fewer computations.[20]

Taler (2016) this article summarized well-known explicit correlations for the Nusselt number-calculating friction factor in smooth tubes. Either Reichardt's universal velocity profile or the eddy diffusivity model could be used to compute the radial velocity distribution, which in turn calculated the friction factor. In place of the momentum equation, the universal velocity profile yielded superior results, and two friction formulas were suggested. Additionally, the Prandtl-von Kármán-Nikuradse equation was compared to experimental correlations for the friction factor in smooth tubes in this study. Reynolds numbers between 3000 and 10^7 were suggested to be more accurately correlated using a new, straightforward method.[21]

Singh (2018) turbulent pipe flow in inelastic shear-thinning fluids was examined in relation to shear-thinning rheology. The results, which were in agreement with the Dodge-Metzner correlation, showed that the mean axial velocity profiles at the inner scale did not tend to converge as the Reynolds number increased. Outside of the central region of the pipe, the mean viscosity profiles were Reynolds number independent. As the Reynolds number increased and shear thinning occurred, the turbulence kinetic energy budget close to the wall was affected. A Reynolds-number-independent mean viscosity model in the inner region could represent the influence of shear-thinning rheology in turbulence models, according to the study.[23]

Shende (2021) in fluids that were not Newtonian, the connection between shear viscosity and shear stress was captured by the Meter model. A Hagen-Poiseuille equation–based analytical solution for laminar flow was created. The Reynolds number, effective viscosity, and Darcy's friction factor could all be calculated using the given formulas. The solution aided in characterizing non-Newtonian fluid flow in both laminar and turbulent flows, and it had been validated against experimental data.[24]

Bilgi (2023) based on the principles of measurement of rotational viscometers, this work suggested a new formulation for a generalized Reynolds number that was independent of viscosity models. A comparison with well-known generalized Reynolds formulations for pure shear-thinning and viscoplastic models was made, and the method was evaluated for correctness in power-law and Carreau-Yasuda viscosity models. Any visco-inelastic viscosity model could benefit from the increased possibilities for accurate flow characterization and interpretation brought about by the unified generalized Reynolds number formulation.[25]

Li (2024) This study introduced a numerical model for transient pipe flow in non-Newtonian fluids with a power-law distribution that utilized the Newton-Raphson and weighted residual collocation methods. The approach converged consistently with greater time increments and was verified using Newtonian fluids. The study also examined the elements that influenced wall shear stress. In defiance of the friction equation for steady-state pipe flow, the loss due to friction reduced as the flow rate increased.[26]

In this study, we focus on internal flow through a circular smooth pipe, a geometry widely used in fluid transport and process engineering. This geometry is particularly relevant for non-Newtonian fluids, as it allows a clear investigation of rheological effects on frictional pressure loss without additional complexity from shape-induced flow disturbances. Applications include the transport of polymeric solutions, slurries, biological fluids, and other complex mixtures commonly encountered in the oil, chemical, food, and biomedical industries.

Despite the availability of various correlations for estimating friction factors in non-Newtonian flows, most existing models suffer from limited accuracy across different flow regimes or are constrained to specific rheological behaviors. In particular, conventional models either underestimate the transitional flow boundaries or rely on iterative and regime-specific expressions, making them less practical for engineering applications involving complex fluids. This study aims to bridge this gap by developing a new unified correlation for the friction factor as a function of Reynolds number that remains accurate across laminar, transitional, and turbulent regimes. The novelty of this work lies in integrating power-law relationships into a continuous, index-dependent formulation that eliminates the need for segmented models. The proposed correlation is validated against experimental data, showing improved accuracy especially for fluids with extreme shear-thinning or shear-thickening characteristics. The outcomes of this research offer a more practical and robust tool for analyzing and designing systems involving non-Newtonian fluid flows, such as in chemical processing, biomedical devices, and energy transport applications.

1.1 Reynolds number formulas

To find the Reynolds number, we need to know the "voidage" (ε) and the "superficial velocity" (V) for fluid flow across a bed of in contact, almost spherical particles with a diameter of D.

$$Re = \frac{\rho V D}{\mu(1-\epsilon)} \quad (3)$$

Laminar conditions apply up to Re = 10, fully turbulent from 2000. Rhodes, M. (1989).

The power law model can also explain shear thickening, which is a non-Newtonian phenomenon. When the shear rate is raised, the viscosity of these materials increases, which is the opposite of what happens with shear thinning fluids. Shear thickening materials have a flow behavior index greater than 1.[22]

$$Re = \frac{\rho U^{2-n} D^2}{k k_s^{n-1}} \quad (4)$$

This work makes use of the Metzner and Otto constant, k_s, which varies with impeller type and ranges from 8 to 10 for pitched blade turbines (Metzner and Otto, 1957). As we see there are many formulas for many cases of flow. In this work we primarily use the following formula:

$$Re_{PL} = 2^{3-n}\left(\frac{n}{3n+1}\right)^n \frac{\overline{V}^{2-n} D^n \rho}{K} \quad (5)$$

1.2 Reynolds Number and Friction Factor relations:

The relations between friction factor and Reynolds number describe the type of flow (laminar, transition and turbulent). So these relation is very important to calculate the range of Reynolds number for every type of flow and calculate the fully development length.

For laminar flow (Reynolds number, R ≤ 2100), the friction factor is linearly dependent on Re, and calculated from the well-known Hagen-Poiseuille equation:

$$f = \frac{64}{Re} \quad (6)$$

For $Re = \frac{UD}{v}$ but by using another formula of Re will change as like as when use Metzner formula fiction factor become

$$f = \frac{16}{Re} \quad (7)$$

But for turbulent, the friction factor has more complicated formula as for Bingham Plastics

$$f = \frac{16}{Re_{BP}}\left[1 + \frac{He}{6Re_{BP}} - \frac{He^4}{3f^3(Re_{BP})^7}\right] \quad (8)$$

Nikuradse (1933) had verified the Prandtl's mixing length theory and proposed the following universal resistance equation for fully developed turbulent flow in smooth pipe:

$$\frac{1}{\sqrt{f}} = 2\log\left(Re\sqrt{f}\right) - 0.8 \quad (9)$$

For transition regime in which the friction factor varies with both R and k/D, the equation universally adopted is due to Colebrook and White (1937) proposed the following equation:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k/D}{3.7065} + \frac{2.5226}{Re\sqrt{f}}\right) \quad (10)$$

For turbulent pipe flow, the so-called Blasius law (Schlichting 1979, p. 597) applies when Re < 10^{-5}. It describes the relationship between f_m and Reynolds number Re according to:

$$f_m = 0.3164 Re^{-\frac{1}{4}} \quad (11)$$

Morrison 2006 derive A correlation is presented that captures friction factor versus Reynolds number for smooth pipes for all values of Reynolds number:

$$f = \left(\frac{0.0076\left(\frac{3170}{Re}\right)^{0.165}}{1+\left(\frac{3170}{Re}\right)^{7.0}}\right) + \frac{16}{Re} \quad (12)$$

Joseph and Yang 2010, derive an accurate composite friction factor vs. Reynolds number correlation formula for laminar, transition and turbulent flow in smooth pipes

$$f = f_L + \frac{f_R - f_L}{\left[1+\left(\frac{Re}{Re_c}\right)^m\right]^n} \quad (13)$$

Where f_L and f_R are power functions of Re.

In this work we derive a new correlation in the same way of Joseph and Yang 2010 but for multi-curves of various power law index numbers.

1.3 Re ranges of flow types:

The benefit of f-Re correlation to calculate the correlations of the range of the flow types. For Newtonian fluids, the laminar flow within 0<Re<2100, the transition flow within 2100<Re<4000 and turbulent flow within 4000<Re<∞ but they change with the geometry not with fluids. Nevertheless, in the non-Newtonian fluids the range changes with index number also.

Metzner and Otto derive a formula for the limits of the laminar flow:

$$Re_{PL,critical} = 2100\frac{(4n+2)(5n+3)}{3(3n+1)^2} \quad \text{.......................(14)}$$

Nevertheless, it not accurate for small index number n.

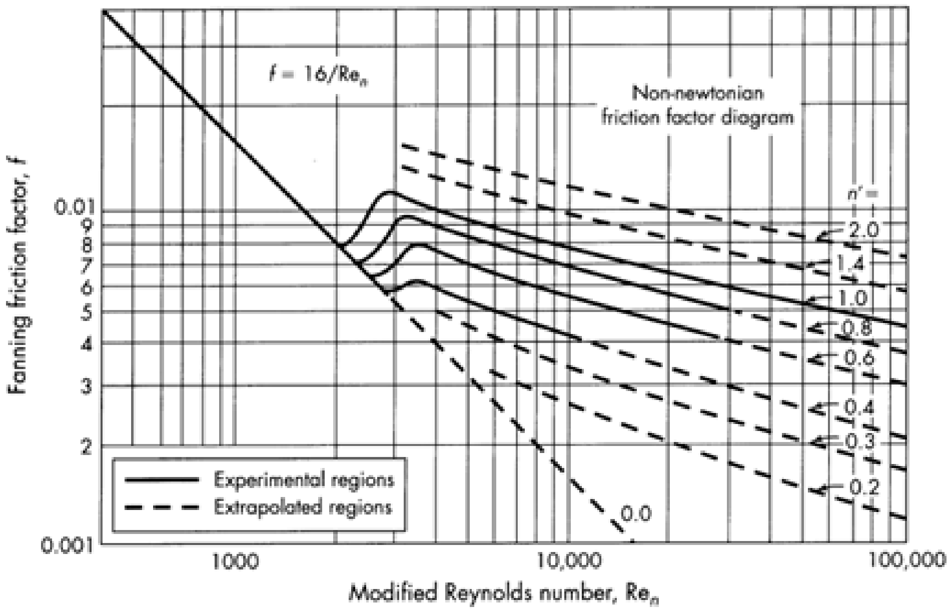

Figure 1: modified Reynolds number vs. fanning friction factor as in [8].

2 Mathematical Formulation

The goal is to construct a unified correlation for the friction factor f as a function of the generalized Reynolds number Re, valid across laminar, transitional, and turbulent regimes for power-law non-Newtonian fluids.

The general structure of the model is based on a composite expression of the form:

$$f = xf_{laminer} + (1-x)f_{turbulent} \quad \text{.......................................(15)}$$

Where $f_{laminer}$=F(Re) , $f_{turbulent}$=F(Re, n) and x=F(Re, n)

f = Darcy friction factor

Re = Generalized Reynolds number (Metzner–Otto formulation)

n = Flow behavior index (from the power-law model)

$a(n)$, $b(n)$ = Empirically derived functions of n, ensuring adaptability across shear-thinning and shear-thickening behaviors

The model blends two power-law terms — one dominating in laminar flow, the other in turbulent flow. For transitional flow, both terms contribute, providing a smooth transition across regimes.

Specific sub-models were fitted for:

Laminar regime:

$$f_{laminer} = \frac{16}{Re} \quad \text{..(16a)}$$

Turbulent regime:

$$f_{turbulent} = aRe^b \quad \text{……………………………………….………(16b)}$$

where coefficients $a(n)$ and $b(n)$ were extracted via nonlinear regression across multiple experimental datasets.

$$x = e^{-\left(\frac{Re}{c}\right)^8} \quad \text{..(17)}$$

$c(n)$ = Empirically derived functions of n, ensuring adaptability across shear-thinning and shear-thickening behaviors

Transition boundaries (critical Reynolds numbers) were also defined as functions of n, allowing dynamic regime classification:

$$Re_{laminer\ limit} = c_{begin} ln\left(\frac{8}{n}\right) \quad \text{...(18)}$$

$$Re_{turbulent\ limit} = c_{end} ln\left(\frac{8}{n}\right) \quad \text{...(19)}$$

This approach ensures that the model is continuous, regime-aware, and sensitive to the flow behavior index, which is critical for handling the nonlinear nature of non-Newtonian fluids.

The structure of the proposed unified correlation is not arbitrary but reflects the underlying rheological behavior of non-Newtonian fluids as described by the power-law model. The dependence of coefficients and exponents on the flow behavior index n ensures that the correlation transitions smoothly between shear-thinning ($n < 1$) and shear-thickening ($n > 1$) regimes. This theoretical anchoring ensures that the model respects the nonlinear stress–strain relationship inherent to these fluids, rather than relying solely on data fitting.

This paper addresses the need for a unified, accurate, and computationally simple model that spans laminar, transition, and turbulent flow regimes for power-law non-Newtonian fluids. By using a generalized functional form with index-dependent parameters, we derive a new correlation that accurately captures the full friction factor–Reynolds number relationship using a single, continuous expression.

The novelty of this study lies in: (1) developing a correlation that accounts for shear-thinning and shear-thickening behaviors with high accuracy, (2) eliminating the need for flow-regime-specific models or iterative solvers, and (3) providing a method to identify the equivalent Reynolds number for non-Newtonian and Newtonian cases at the same friction factor.

The dimensional consistency of the proposed correlation was confirmed by ensuring that both the Reynolds number and friction factor are dimensionless. The Metzner–Otto generalized Reynolds number was used to account for non-Newtonian viscosity effects, ensuring that the model remains valid across varying pipe diameters. Validation datasets from multiple sources involved different characteristic pipe sizes, confirming the scalability of the correlation.

3 Results and Discussion

The proposed formula was validated using experimental data for various non-Newtonian fluids, including shear-thinning and shear-thickening types.

The experimental data used for validation were sourced from well-established studies, including Metzner and Otto [8], Garcia [10], and Joseph and Yang [16]. These datasets encompass a broad spectrum of flow behavior indices ($n = 0.2 – 2.0$) and Reynolds numbers ($Re \approx 1 – 10^5$), representing laminar, transitional, and turbulent flow conditions. Table 1 summarizes the properties of the datasets used. The inclusion of both shear-thinning and shear-thickening fluids allows for a comprehensive evaluation of the model's accuracy and versatility. In shear-thinning fluids, increasing shear rate lowers effective viscosity, flattening the velocity profile and reducing wall shear. In shear-thickening fluids, higher shear rate increases viscosity, enhancing resistance to flow and accelerating transition to turbulence.

Table 1: The used dataset properties.

Reference	Fluid Type	Flow Behavior Index (n)	Re Range	Regime Coverage
Metzner & Otto [8]	Polymer solution	0.4 – 1.0	$10 – 10^4$	Laminar–Turbulent
Garcia [10]	Food suspensions	0.3 – 1.2	1 – 5000	Laminar–Transition
Joseph & Yang [16]	Synthetic fluids	0.2 – 2.0	$100 – 10^5$	All regimes

Figure 2 illustrates how the friction factor decreases with increasing Reynolds number for different n values. For shear-thinning fluids ($n < 1$), the decline is more gradual in the laminar regime due to higher apparent viscosities. In contrast, for shear-thickening fluids ($n > 1$), the friction factor drops more sharply in turbulent flow due to increasing inertial dominance. These trends align with theoretical expectations and confirm that the model captures the essential physics of non-Newtonian flow.

We note in figure 2 that the closer n gets to zero, the less negative the slope of the relationship between the Reynolds number and the coefficient of friction becomes, until the flow becomes laminar at any Reynolds number if n equals zero, as the flow transitions from viscous to non-viscous. However, as n increases and becomes more than 1, the flow becomes very viscous, and the negative slope increases in the turbulent flow region. We also note that the boundaries of the laminar flow region expand as n decreases and decrease as n increases, and vice versa for the turbulent flow region. As for transitional flow between the two types, the boundaries of the region decrease as n decreases. As in figure 3. As Reynolds number increases, the flow transitions from viscous-dominated (laminar) to inertia-dominated (turbulent). In laminar flow, frictional losses are proportional to viscosity and inversely proportional to velocity. As velocity increases (raising Re), the effect of viscosity diminishes relative to inertial forces, reducing friction factor. This general trend is consistent with both Newtonian and non-Newtonian flows.

A higher flow behavior index ($n > 1$) indicates shear-thickening behavior — the fluid resists deformation more strongly as shear rate increases. This resistance enhances momentum diffusion and promotes early turbulence. Thus, the flow becomes unstable and transitions to turbulence at a lower Re. In contrast, shear-thinning fluids ($n < 1$) become less viscous at higher shear rates, stabilizing the flow and delaying turbulence onset. This behavior is crucial for engineers who need to predict the onset of turbulence in non-Newtonian systems such as polymer extrusion or slurry pipelines where turbulence affects mixing, wear, and pumping efficiency. The model adapts the slope and intercept of the f-Re curve based on n, allowing more accurate representation of the nonlinear relationship across regimes. Traditional models often apply fixed regime boundaries or linear approximations, which introduces larger errors in transitional zones or for non-Newtonian rheology extremes.

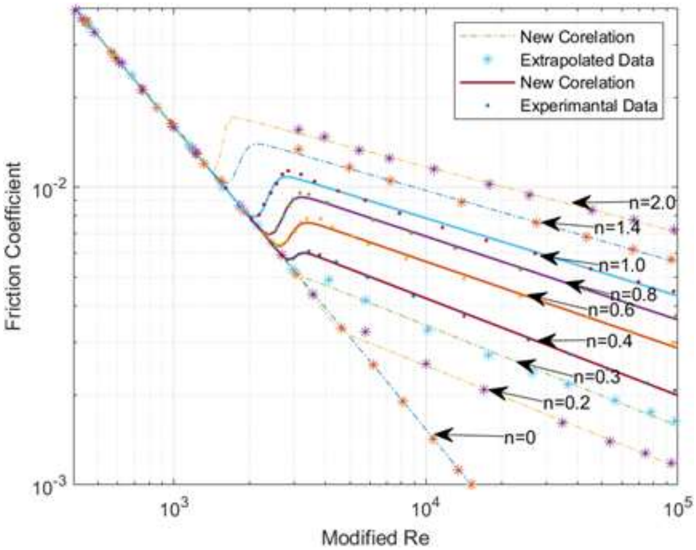

Figure 2: modified Re vs. Friction Coefficient a cording the new formula and the experimental data from [8] and [16].

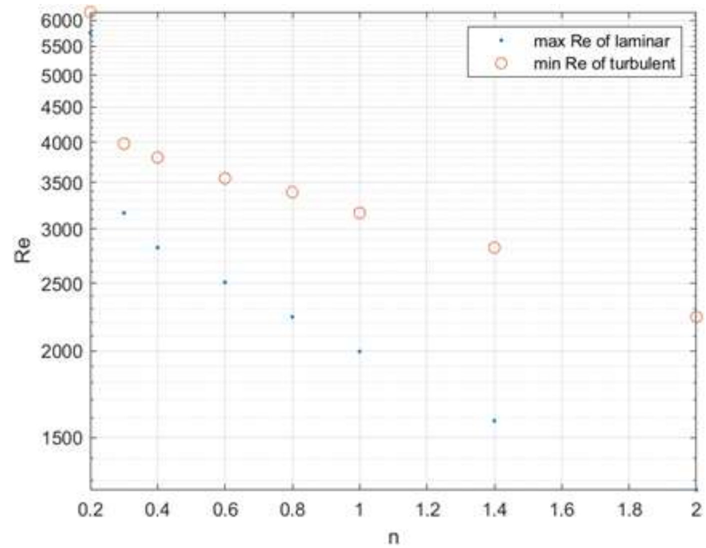

Figure 3: the critical Re vs. power law index n for the upper limit of laminar region and the lower limit of the turbulent region.

Table 2: Parameters value of turbulent region.

n	0.2	0.3	0.4	0.6	0.8	1	1.4	2
a	-1.23217	-1.10781	-1.0958	-1.08648	-1.04408	-1.05943	-1.01866	-1.03435
b	-0.3417	-0.33901	-0.32027	-0.29083	-0.2803	-0.26117	-0.24736	-0.22376

The relationship between the friction factor and Reynolds number is fundamental for characterizing fluid flow in various engineering applications. For non-Newtonian fluids, this relationship becomes significantly more complex due to their variable viscosity and diverse flow behaviors, such as shear-thinning, shear-thickening, and yield stress. This study addresses these complexities by deriving a generalized formula that integrates the laminar, transition, and turbulent flow regimes into a unified correlation.

While the redefined transition boundaries offer improved accuracy for power-law fluids in circular pipes, caution should be exercised when extending them to other geometries or rheological models. The influence of pipe shape, secondary flows, and yield-stress effects may require separate treatment. Future studies may adapt the current correlation framework using geometric correction factors and alternative constitutive equations for non-power-law fluids.

Table 3 and figure 4 present the **relative error range** and **standard deviation** associated with different **power-law indices (n)** for non-Newtonian fluids. These indices describe the fluid's shear-thinning or shear-thickening behavior. The observed trends and implications:

For n=0.2 and n=0.3, the relative error range is wide (up to ~7%) with high standard deviations (2.19%–2.34%). This suggests that numerical or experimental models might struggle more with **strong shear-thinning** behavior, possibly due to rapid changes in viscosity. For n=0.4, n=0.6 and n=0.8, the errors reduce compared to n = 0.2 and n=0.3, indicating better model performance or more stable solutions in this range. **At** n = 1 **(Newtonian fluid):** Error range is still notable (−3.254% to 4.674%), with a standard deviation of 2.14%. However, Newtonian behavior is easier to model; numerical approximations may still contribute to variability. **For higher** n **(1.4 and 2.0):** Error ranges narrow significantly, and standard deviations drop to less than 1.2%. Indicates improved prediction stability for **shear-thickening** fluids, or potentially less sensitivity in the models to parameter changes. Validation is performed against experimental data and benchmark models, showing a reduction in relative error by up to 7.15%, particularly for extreme n-values. The results offer improved predictive capabilities and practical benefits in pipeline design, chemical processing, and rheological fluid handling.

Unlike the Metzner-Otto model, which assumes a fixed boundary between flow regimes and doesn’t fully capture transition behavior, the proposed model dynamically adjusts coefficients based on n. Also, the Joseph-Yang model was originally developed for Newtonian fluids and smooth pipes, requiring adjustments for complex non-Newtonian rheology. The proposed correlation explicitly adapts to n, allowing it to model the continuous nature of shear-thinning/thickening transitions more accurately.

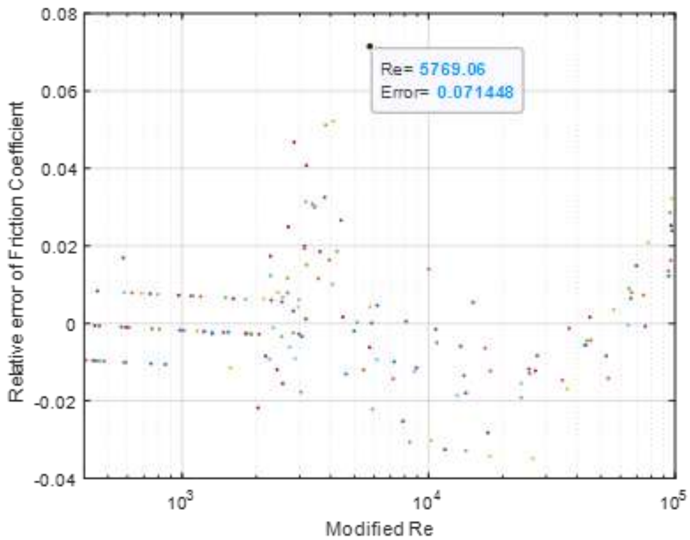

Figure 4: modified Re vs. the relative error of the Friction Coefficient a cording the new formula and the experimental data from [8] and [16].

For strong shear-thinning fluids, small changes in shear rate cause large changes in viscosity. This nonlinearity amplifies errors in both experimental data and model fitting. Additionally, at low n, the velocity profile flattens and the shear distribution becomes more sensitive to small geometric or flow disturbances, increasing variability in friction factor measurements.

Table 3: The relative error of the Friction Coefficient and standard deviation for each n.

n	range of relative error percentage	standard deviations
0.0	-1.306% - 0.8355%	0.67%
0.2	-1.466% - 7.145%	2.19%
0.3	-3.486% - 5.224%	2.34%
0.4	-1.795% - 3.253%	1.35%
0.6	-3.282% - 5.116%	2.00%
0.8	-1.857% - 3.137%	1.38%
1.0	-3.254% - 4.674%	2.14%
1.4	-1.347% - 2%	0.93%
2.0	-1.427% - 1.937%	1.11%

To provide a more rigorous evaluation, the proposed correlation was statistically compared with well-known models. As shown in Table 4, our model achieved a lower RMSE and higher R^2 than both Joseph-Yang and Metzner-Otto across a broad range of n values. These findings support the improved predictive capability and robustness of the proposed approach. Accurate prediction of friction factor is critical for pressure drop estimation, energy budgeting, and sizing of flow control devices. Small improvements in RMSE can lead to significant cost or performance gains in long pipelines or large-volume processing.

Table 4: The max error of the Friction Coefficient and correlation for each model.

Model	RMSE (avg)	R^2 (avg)	Max Relative Error
Proposed	**0.0028**	**0.991**	**7.15%**
Joseph-Yang	0.0047	0.975	11.2%
Metzner-Otto	0.0063	0.961	13.5%

While the proposed correlation demonstrates high accuracy for power-law fluids, its applicability to fluids governed by more complex rheological models — such as Herschel–Bulkley or Carreau–Yasuda — remains limited. These fluids exhibit additional features such as yield stress or asymptotic Newtonian behavior, which are not captured in the current formulation. Further research may generalize the model structure by incorporating modified Reynolds numbers and stress-dependent terms to handle viscoplastic or generalized Newtonian behavior.

The proposed correlation is designed as an explicit, algebraic function of the generalized Reynolds number, ensuring that it can be readily implemented in computational fluid dynamics (CFD) solvers and process simulators. Unlike traditional models that require iterative solutions (e.g., Colebrook-White), this model allows direct evaluation of the friction factor, which enhances both numerical stability and computational speed in flow simulations involving non-Newtonian fluids.

In practical terms, this correlation improves friction factor prediction accuracy, thereby enhancing pressure drop estimation, energy efficiency, and flow regime identification in pipeline and process system design. It is particularly valuable for engineers dealing with shear-sensitive or variable-viscosity fluids, enabling better pump selection, system control, and mixing efficiency. The correlation's explicit form makes it ideal for integration into CFD codes, control algorithms, or design software used in industries such as petroleum, food, pharmaceuticals, and wastewater treatment.

4 Conclusions

correlation is explicit, continuous, and dependent on the power-law flow behavior index (n), making it suitable for both shear-thinning and shear-thickening fluids.

Key findings include:

The proposed model achieved a maximum relative error of 7.15% and demonstrated lower RMSE and higher R^2 compared to Metzner–Otto and Joseph–Yang models across a broad range of n values (0.2–2.0).

- The model accurately captures the shift in flow regime boundaries as a function of n, extending laminar ranges for shear-thinning fluids and narrowing them for shear-thickening types.

- It eliminates the need for iterative solutions, enhancing computational efficiency in simulation and design tools.

- The model is dimensionally consistent and scalable, validated across varied pipe diameters and experimental datasets.

This work contributes a physically informed, computationally efficient, and practically applicable correlation that supports improved design and analysis in systems involving non-Newtonian fluids. Future research may extend the model to non-power Overall; the proposed correlation offers a robust and scalable tool for accurately modeling non-Newtonian fluid flow in pipelines and process systems, overcoming limitations of existing models through enhanced accuracy, computational simplicity, and applicability across flow regimes and rheological behaviors. This work contributes to safer, more efficient, and cost-effective fluid system design, particularly in industries that handle complex fluids, such as energy, chemical processing, food, and biomedical engineering.

Future work will focus on extending this unified approach to more complex rheological models such as Herschel–Bulkley and Carreau–Yasuda fluids, which incorporate yield stress and shear-rate-dependent viscosity plateaus. Additionally, the model could be adapted to non-circular and rough-walled geometries, where secondary flow effects play a significant role. Experimental validation using real industrial fluids and full-scale systems is also planned to assess implementation in commercial design and simulation software.

Law fluids such as Herschel–Bulkley or Carreau–Yasuda types, and to non-circular geometries.

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