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Original Article

Deep Nonnegative Matrix Factorization for Fair and Explainable Graph Clustering

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ABSTRACT

In Graph clustering has been widely applied to a variety of domains including but not limited to, social networks, health care, and education, but existent methods largely overlook two important challenges: fairness regarding sensitive attributes and interpretability of the obtained communities. In this work, we present DFEGC (Deep Fair Explainable Graph Clustering), a fresh framework that makes a balance for both accuracy, fairness and interpretability. DFEGC uses an N-layer NMF backbone to discover hierarchical graph structure, integrates a Hilbert-Schmidt Independence Criterion (HSIC)-based fairness regularizer to prevent discrimination in unsupervised clustering, and includes explainability modules to promote sparsity and semantic saliency when learning latent factors. Extensive experiments on six real datasets show that DFEGC can consistently outperform state-of-the-art baselines, with up to 15-20% improvements in NMI and ARI, and reduce the statistical parity difference and the equal opportunity difference by large margins. Besides, our model results in sparser and semantically more coherent latent representations, thus improving the interpretability of clusters. These findings demonstrate that DFEGC is a principled and affordable approach to responsible graph clustering, and strikes a balance between clustering accuracy and fairness. Its explainability also makes it appropriate for high-stakes applications when trust and fairness are crucial.

Keywords: graph clustering, fairness, explainability, nonnegative matrix factorization, deep learning.

Introduction

Graph clustering has relevance in numerous domains such as social networks, bioinformatics, and recommender systems. Recent surveys [1] have highlighted its critical role in uncovering community structures through deep learning methods such as GNN-based DMoN pooling [2]. Conceptually, graph clustering attempts to identify groups of nodes that might be structurally or semantically similar. The traditional approaches for graph clustering, such as spectral clustering [3] and modularity maximization [4] assume strong linear relations, complemented by determined handcrafted metrics of similarity that limit their ability to model the complex patterns and multi-scale relationships found inside real-world graphs [5].

With the recent developments of deep learning methods, such as Graph Neural Networks (GNNs) [6, 7], advances have also been seen in graph representation learning. Yet, techniques that merge deep NMF with representation learning such as contrastive deep NMF have demonstrated improved hierarchical and attribute-aware clustering [8]. Meanwhile, in the other expected horizon, NMF came forth as an interpretable and very efficient approach to clustering graphs [9]. Deep NMF is a multi-layered extension of the original NMF approach [10], intended to offer more representation flexibility while sustaining the interpretability of classic NMF.

The notion of fairness, paramount in clustering, has largely remained eclipsed from past research. Graph clustering techniques are increasingly constituted within decision-making systems e.g., for instance, fairness-aware community detection through semidefinite relaxation has risen in recent years enabling explicit trade-offs between clustering quality and demographic fairness [11]. It becomes crucial to guarantee that lineage review clustering outcomes do not systematically disadvantage some demographics [12, 13]. In clustering, attributes like gender, race, or age may taint the process, generating bias, more so in networks with imbalanced or homophilic structures.

Fairness has been put forth as a domain rife with studies in classification; however, comprehensive surveys on fairness in graph learning [14] emphasize that most efforts focus on supervised tasks like node classification or link prediction; fair unsupervised methods such as fair clustering are still underrepresented [15]. Early attempts such as Fair-NMF manifest the promise of fairness-aware matrix factorization. More

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recently, iFairNMTF introduced contrastive regularization for individual fairness in graph clustering [16], offering flexible accuracy-fairness trade-offs. Nonetheless, these approaches are still limited and do not capture deeper hierarchical patterns via deep NMF.

Contributions. In this paper, we address the fairness issue in deep NMF-based graph clustering by:

- Proposing a novel fairness-aware deep NMF framework for graph clustering.
- Embedding group fairness constraints directly into the multi-layer NMF optimization process.
- Evaluating trade-offs between clustering performance and fairness using real-world datasets and fairness metrics such as demographic parity and equality of opportunity.

Our method enhances the interpretability and expressiveness of deep NMF while mitigating bias, offering a principled path forward for equitable and effective graph clustering.

Related Work

Graph Clustering

The goal of graph clustering is to divide a graph's nodes into logical clusters. Spectral clustering [3] and modularity optimization [4] are examples of classical techniques that use modularity scores or graph Laplacian eigenvectors to identify community structures. However, these methods often assume linear relationships and struggle to scale with large and noisy networks [2].

Through representation learning, deep learning has brought graph clustering back to life. By combining neighborhood data, GNN-based models such as GCN [6] and GraphSAGE [20] learn node embeddings. These learned embeddings are subjected to clustering, which frequently produces better results than conventional methods [21].

Differentiable pooling techniques, such as DiffPool [25], were introduced recently for hierarchical graph clustering, allowing for end-to-end optimization and soft assignments. A modularity-inspired unsupervised clustering objective within GNNs was proposed more recently by Deep Modularity Networks (DMoN) [1], which demonstrated notable gains in cluster recovery. Robust end-to-end clustering models are necessary, as demonstrated by the limitations of many pooling techniques across noisy domains revealed by graph pooling benchmarks [26].

Nonnegative Matrix Factorization and Deep Variants

The main term for NMF in relation to clustering is its ability to factor an input matrix into features that are part-based and of lower dimension [9]. With regard to graphs, adjacency and feature matrices can be factored into node embeddings that can be used to detect latent community structures.

The Deep NMF (DNMF) models improve the representation power of shallow NMF by stacking multiple layers of factorizations, and retain the interpretability of shallow models [10]. The effectiveness of Deep NMF has also been demonstrated in recent works in the domains of image analysis, and recommender systems, in addition to clustering [22]. However, such models tend to ignore the notion of fairness and bias in favor of clustering accuracy.

The Contrastive Deep NMF (CDNMF) framework [8] is the only one that integrates deep NMF with community detection, as well as contrastive learning, effectively utilizing the graph structure as well as node attributes. The multi-constraint NMF variants based on orthogonal constraints [27] further enhance the community detection results by enforcing structural regularization, which in turn improves the structural constraints.

Fairness in Machine Learning and Graph Clustering

The notion of fairness in artificial intelligence is in regard to the absence of prejudice in model results related to sensitive factors like gender or race. The results of a model should be free of bias in relation to sensitive factors such as gender or ethnicity [13, 18]. In graph-structured data, bias can emerge from structural homophily or skewed representation, which results in inequitable outcomes in subsequent tasks [12].

Methods for fair graph clustering include FairWalk [17], a preprocessing method that equalizes random walks over demographic groups, and FairGNN [23] and FairDrop [24], which are in-processing methods that change model objectives to lower demographic disparity. These methods are predominantly based on GNNs and have little extension to matrix factorization.

Graph contrastive learning approaches, such as CSGCL [28], introduce community-strength-aware augmentations to maintain the community structure in embeddings. Other methods, such as CEGCL [29], tackle community-aware contrastive learning with self-training to mitigate unfairness in community detection tasks.

Fairness-aware Matrix Factorization

There have been few but encouraging recent attempts to add fairness to matrix factorization. Through regularization, Fair-NMF [19] aligns latent representations across demographic groups. Fairness is enhanced, but only for superficial models. This paper attempts to close the gap in the literature by explicitly integrating fairness into deep NMF for graph clustering.

Despite using contrastive regularization to enhance individual-level fairness in NMF, iFairNMTF [16] is not applicable in graph-specific clustering scenarios.

Related Work comparison

In order to summarize existing works in a structured manner, Table 1 lists and compares classical graph clustering, NMF and its deep variants, fairness in machine learning and graph clustering, and fairness-aware matrix factorization. The table shows deep graph clustering models such as GCN [6],

GraphSAGE [20], and DMoN [1] outperform classical models through representation learning; however, they lack fairness considerations. Similarly, NMF [9,10,22,27] balances interpretability and effectiveness and remains strong for clustering, though it ignores fairness. While fairness-focused approaches such as FairWalk [17], FairGNN [23], and FairDrop [24] enhance demographic balance, they operate on embeddings and GNNs, leaving matrix factorizations untouched. To that effect, uniquely Fair-NMF [19] and iFairNMTF [16] approach fairness in factorization but operate in shallow factorizations and leave hierarchical deep factorizations uncovered. This showcases a clear gap in the research, as demonstrated by Table 1: no existing work explicitly integrates fairness constraints into deep NMF for graph clustering, which motivates the contributions of this study.

Table 1: Comparison Table Based on Your Related Work.

Category	Method / Work	Approach	Strengths	Limitations	Year
Graph Clustering	Spectral Clustering [3]	Eigen-decomposition of Laplacian	Well-studied, strong theoretical foundation	Poor scalability, assumes linear relations	2002
	Modularity Optimization [4]	Modularity maximization	Captures community structures intuitively	Resolution limit, not scalable	2006
	GCN [6]	GNN-based node embeddings	Learns from neighborhood structure	Supervised setting, limited clustering focus	2017
	GraphSAGE [20]	Inductive GNN embeddings	Scales to large graphs	Embeddings not fairness-aware	2017
	Deep Graph Infomax [21]	Contrastive embedding learning	Effective unsupervised embeddings	Sensitive to augmentation strategy	2019
	DMoN [1]	Modularity-inspired pooling in GNNs	Improves unsupervised clustering	Still limited fairness considerations	2023
NMF and Deep Variants	NMF [9]	Matrix factorization	Interpretable, simple	Shallow, limited expressiveness	2009
	Deep Matrix Factorization [10]	Stacked NMF layers	Captures deeper structure	Less scalable, fairness not addressed	2017
	Deep NMF for Co-Clustering [22]	Multi-modal NMF	Handles text/visual data jointly	Focused on data, not fairness	2022
	Contrastive Deep NMF (CDNMF) [8]	Contrastive learning + NMF	Uses graph + attributes jointly	Fairness not integrated	2023
	Multi-Constraint NMF [27]	Orthogonal + sparse regularization	Improves community detection	Limited evaluation on fairness	2024
Fairness in ML & Graph Clustering	FairWalk [17]	Preprocessing: fair random walks	Simple, scalable	Only embedding-level fairness	2019
	Equality of Opportunity [18]	Supervised fairness framework	Strong fairness guarantee	Not for clustering	2016
	FairGNN [23]	Sensitive-attribute-aware GNN	Improves demographic fairness	Task-specific, not clustering-focused	2021
	FairDrop [24]	Edge dropout for fairness	Easy to integrate, improves fairness	Reduces graph connectivity	2022
	Fairness Surveys [14,15]	Comprehensive reviews	Taxonomy of fairness methods	Highlight gaps in unsupervised clustering	2023–24
Fairness-aware Matrix Factorization	Fair-NMF [19]	Regularized NMF	Aligns latent space across groups	Limited to shallow models	2023
	iFairNMTF [16]	Contrastive fairness regularization in NMF	Individual-level fairness	Not applied to graph clustering	2024

To date, no work explicitly integrates fairness into deep NMF for graph clustering—a gap this paper aims to fill.

Proposed Method

In this section, we introduce our novel model: **Deep Fair and Explainable Graph Clustering (DFEGC)**. It integrates deep nonnegative matrix factorization with fairness constraints and interpretability mechanisms, suitable for both attributed and plain graphs. The core idea is to construct a multi-layer representation while embedding fairness regularization and explainability signals into the learning process.

Problem Definition

Let $G = (V, E)$ be an undirected graph with $|V| = n$ nodes and $|E| = m$ edges. Let $X \in \mathbb{R}^{n \times d}$ be the node attribute matrix and

$A \in \mathbb{R}^{n \times n}$ be the adjacency matrix. Given X and A, our goal is to partition the nodes into k clusters $\{C_1, \ldots, C_k\}$ such that:

- Nodes in the same cluster are structurally and semantically similar.
- The clustering respects fairness with regard to sensitive attribute(s) S (e.g., gender, race).
- The learned factors allow interpretable insights into clustering outcomes.

Deep Nonnegative Representation Learning

We employ a deep, layer-wise nonnegative matrix factorization (NMF) model to construct hierarchical representations. Given X, the model factorizes it through L layers as:

$$\mathcal{L}_{\text{Laplacian}} = \text{Tr}\left(H^{(L)} L H^{(L)T}\right) \quad (1)$$

where:

- $W^{(l)} \in \mathbb{R}_{+}^{r_{l-1} \times r_l}$: basis matrix at layer l ($r_0 = d$)
- $H^{(L)} \in \mathbb{R}_{+}^{r_L \times n}$: final encoding used for clustering

Each layer learns a compressed representation that retains nonnegativity, interpretability, and modularity. To facilitate structure preservation, we incorporate graph regularization using the graph Laplacian $L = D - A$:

$$\mathcal{L}_{\text{Laplacian}} = \text{Tr}\left(H^{(L)} L H^{(L)T}\right) \quad (2)$$

where D is the degree matrix and A is the adjacency matrix. For an embedding matrix $H^{(L)}$.

Expanding the trace yields:

$$\mathcal{L}_{\text{Laplacian}} = \text{Tr}\left(H^{(L)} L H^{(L)T}\right) = \left(\frac{1}{2}\right) \sum_{i,j} A_{ij} \parallel \text{H}_i - \text{H}_j \parallel^2 \quad (3)$$

where H_i and H_j are row vectors of $H^{(L)}$. Minimizing this term enforces smoothness by encouraging adjacent nodes to have similar embeddings.

Fairness Regularization with HSIC

To reduce dependency between cluster assignments and sensitive attributes S, we employ the Hilbert-Schmidt Independence Criterion (HSIC). HSIC measures statistical dependence between two variables in a Reproducing Kernel Hilbert Space (RKHS):

$$\text{HSIC}\left(H^{(L)}, S\right) = \frac{1}{(n-1)^2} \text{Tr}(K_H H K_S H) \quad (4)$$

where K_H and K_S are kernel matrices for $H^{(L)}$ and S respectively.

If $H^{(L)}$ and S are independent, the expectation of HSIC approaches 0. Thus, minimizing:

$$\mathcal{L}_{\text{fair}} = \lambda_f \cdot \text{HSIC}\left(H^{(L)}, S\right) \quad (5)$$

enforces independence and ensures fairness. While we focus here on the methodological formulation, a formal discussion of the synergy between HSIC and Laplacian regularization can be found in **Appendix A**

Explainability via Basis Alignment

Interpretability is facilitated by aligning the basis vectors in $W^{(L)}$ with semantically meaningful patterns in the data. Inspired by topic models, each basis vector can be interpreted as a "concept" or "theme":

- We constrain $W^{(L)}$ to be sparse and distinct (orthogonality penalty)
- Encourage alignment with node features by maximizing covariance with selected feature groups

The objective is:

$$\mathcal{L}_{\text{exp}} = \lambda_e \left(\parallel W^{(L)} \parallel_1 - \alpha \cdot \text{cov}\left(W^{(L)}, X\right) \right) \quad (6)$$

The ℓ_1 norm promotes sparsity, which improves interpretability, while the covariance term aligns $W^{(L)}$ with input features X. Together, these ensure sparse yet semantically coherent basis vectors.

Overall Objective

The joint loss combines reconstruction, graph structure preservation, fairness, and explainability:

$$\mathcal{L} = \mathcal{L}_{\text{recon}} + \gamma \cdot \mathcal{L}_{\text{Laplacian}} + \lambda_f \cdot \mathcal{L}_{\text{fair}} + \lambda_e \cdot \mathcal{L}_{\text{exp}} \quad (7)$$

where:

- $\mathcal{L}_{\text{recon}} = \parallel X - W^{(1)} \ldots W^{(L)} H^{(L)} \parallel_F^2$
- $\gamma, \lambda_f, \lambda_e$: trade-off hyperparameters

And each term contributes to a distinct property:

- $\mathcal{L}_{\text{recon}}$: ensures reconstruction accuracy.
- $\mathcal{L}_{\text{Laplacian}}$: enforces graph structural smoothness.
- $\mathcal{L}_{\text{fair}}$: reduces bias with fairness constraints.
- $\mathcal{L}_{\text{exp}}$: enhances interpretability.

Since each component is convex (or can be relaxed to a convex form), their weighted combination yields a tractable optimization problem balancing accuracy, fairness, and explainability.

Model Complexity and Time Analysis

To evaluate how well our Deep Fair and Explainable Graph Clustering (DFEGC) model scales and performs, we analyze its **computational complexity** during both training and inference.

Time Complexity

We begin by defining some key variables: let n be the number of nodes, d the input feature dimension, k the number of clusters, L the number of layers, and t the number of training iterations per layer.

The first major component is the layer-wise pretraining phase using Nonnegative Matrix Factorization (NMF). At each layer, the input matrix $X^{(l)} \in \mathbb{R}^{n \times r_l}$ is factorized into two matrices, $W^{(l)}$ and $H^{(l)}$, using multiplicative updates. Each iteration has a time complexity of $\mathcal{O}(nr_{l+1}r_l)$, and across all layers and training iterations, the total cost is approximately $\mathcal{O}(Ltnr_{\max}^2)$, where $r_{\max}$ is the largest latent dimension used.

Next, we incorporate graph Laplacian regularization to enforce smoothness in the learned representations. This involves computing the trace term $\mathrm{Tr}(H^\top LH)$, where L is the graph Laplacian. The dominant operations here are a sparse matrix multiplication with complexity $\mathcal{O}(er_L)$—where e is the number of edges—and the trace computation, which adds $\mathcal{O}(nr_L)$, leading to a total cost of $\mathcal{O}(er_L + nr_L)$.

To enforce fairness, we use the Hilbert-Schmidt Independence Criterion (HSIC), which requires computing two kernel matrices, K_H and K_S, both of size $n \times n$. Constructing these kernels has a cost of $\mathcal{O}(n^2 r_L)$ and $\mathcal{O}(n^2 d_s)$ respectively, where d_s is the dimension of the sensitive attribute. The HSIC computation itself, including matrix multiplications and a trace operation, can take up to $\mathcal{O}(n^3)$ in the worst case, making it the most computationally expensive part of the model on large graphs.

Finally, for explainability, we include two operations: computing the ℓ_1 norm of certain matrices, which takes $\mathcal{O}(nr_L)$, and estimating covariance, which requires $\mathcal{O}(nr_L d)$. These operations are relatively lightweight compared to the fairness term.

In summary, while most parts of the model are efficiently scalable, the fairness objective based on HSIC becomes a bottleneck for large graphs. To mitigate this, we adopt low-rank approximations of kernel matrices and use mini-batch training, significantly reducing both memory usage and runtime. These enhancements allow DFEGC to remain both fair and practical for large-scale graph clustering tasks.

Experiments

Experimental Setup

Datasets: We evaluate our proposed DFEGC model on several widely used attributed graph datasets to assess both clustering performance and fairness.

WebKB is a real-world dataset consisting of webpages from computer science departments of various universities (e.g., Cornell, Texas, and Wisconsin). In this dataset, each node corresponds to a webpage, and edges represent hyperlinks between them. Each node is associated with a high-dimensional feature vector derived from its textual content, and labeled into categories such as *course*, *faculty*, *project*, and *student*. We treat the university domain as a sensitive attribute for evaluating fairness in clustering outcomes.

To demonstrate the generalizability of our method, we also conduct experiments on standard citation network datasets: **Cora**, **Citeseer**, and **PubMed**. In these datasets, nodes represent documents and edges indicate citation links. Each node is described by a bag-of-words feature vector and assigned a ground truth topic label. These datasets differ in terms of size, feature dimensionality, and class distribution, enabling a comprehensive evaluation of our model's effectiveness and robustness.

Table 1: Statistics of the datasets used in our experiments.

Dataset	#Nodes	#Edges	#Features	#Classes
WebKB (Cornell)	195	304	1703	5
WebKB (Texas)	187	309	1703	5
WebKB (Wisconsin)	265	515	1703	5
Cora	2708	5429	1433	7
Citeseer	3327	4732	3703	6
PubMed	19717	44338	500	3

Preprocessing: Each graph is symmetrized and normalized. Node features are normalized to unit norm. For sensitive attribute evaluation, we extract domain-specific metadata (e.g., university label in WebKB, author gender in synthetic versions of Cora).

Baselines: We compare DFEGC with several state-of-the-art clustering and fair learning baselines. Each baseline provides a relevant comparison point in terms of clustering quality, fairness, or model complexity:

- **K-Means on Raw Features:** A classical clustering algorithm applied directly to the node feature vectors without considering graph structure. This provides a simple but strong non-graph baseline.
- **Standard NMF:** Nonnegative Matrix Factorization performed on the attribute matrix without incorporating fairness or structural information. It offers a low-rank approximation that reveals latent factors but lacks fairness-awareness.
- **Graph Spectral Clustering:** A widely used method that leverages the graph Laplacian to embed nodes into a lower-dimensional space before applying clustering. It captures structural properties but ignores node attributes and fairness concerns.
- **Fair NMF**: A fairness-aware NMF model that incorporates constraints to reduce disparate treatment or impact based on sensitive attributes. It serves as a representative method for interpretable fair clustering.
- **Sparse NMF:** An NMF variant that promotes sparsity in the factorized matrices to enhance **interpretability** and feature selection. While more explainable, it does not explicitly address fairness or graph structure.

- **Deep NMF (without fairness or explainability):** A deep learning-based extension of NMF that models nonlinear hierarchical representations but lacks fairness constraints and explainability mechanisms, offering a comparison point for model complexity.

Evaluation Metrics: To evaluate the effectiveness of our proposed DFEGC model, we employ a comprehensive set of metrics that cover clustering quality, fairness, and explainability:

- Clustering Performance:
  - **Normalized Mutual Information (NMI):** Measures the mutual dependence between predicted clusters and ground-truth labels, normalized to ensure values between 0 and 1. Higher values indicate better alignment.
  - **Adjusted Rand Index (ARI):** Evaluates the similarity between the predicted and true clustering, adjusted for chance grouping. An ARI of 1 indicates perfect clustering.
- Fairness:
  - **Statistical Parity Difference (SPD):** Captures the difference in cluster assignment probabilities across different sensitive groups. A value close to 0 indicates fairness.
  - **Equal Opportunity Difference (EOD):** Measures the difference in true positive rates between groups defined by sensitive attributes. Lower values denote fairer treatment across groups.
- Explainability:
  - **Sparsity of Basis Matrix ($W^{(L)}$):** Indicates how many input features significantly contribute to the latent representation. Measured by the L_0 norm, higher sparsity suggests better interpretability.
  - **Feature Importance Scores:** Calculated as the average absolute correlation between original input features and the final learned basis. Higher correlations indicate more clearly interpretable contributions from features.

Implementation Details: The model is implemented in PyTorch. Optimization uses Adam with learning rate 0.001. We set number of layers $L = 3$, and use ReLU activation to ensure nonnegativity. Hyperparameters are selected via grid search on a validation set: $\lambda_f \in \{0.01,0.1,1\}$, $\lambda_e \in \{0.1,0.5,1\}$, and $\gamma \in \{0.1,1,10\}$.

Results and Analysis

Clustering Performance Analysis

The clustering performance of various models is evaluated using two metrics: Normalized Mutual Information (NMI, Table 2) and Adjusted Rand Index (ARI, Table 3). Our proposed method, DFEGC, achieves the highest scores on most datasets, demonstrating superior clustering consistency. While traditional methods like K-Means and Standard NMF underperform, specialized variants such as Sparse NMF and Graph Spectral Clustering show competitive results on specific datasets (e.g., Sparse NMF on PubMed). However, DFEGC's robustness across multiple benchmarks highlights its effectiveness in capturing complex data structures. The only exception is the Wisconsin dataset, where Graph Spectral Clustering slightly outperforms DFEGC in NMI, suggesting a potential area for future improvement.

Table 2: Normalized Mutual Information (NMI) Scores.

Model	Cornell	Texas	Wisconsin	Cora	Citeseer	PubMed
K-Means (Raw Features)	0.0429	0.0684	0.0626	0.0922	0.0885	0.5649
Standard NMF	0.1339	0.1470	0.0788	0.2725	0.1157	0.5741
Graph Spectral Clustering	0.1628	0.1549	0.1345	0.2598	0.1259	0.4932
Fair NMF	0.1452	0.1457	0.0680	0.3458	0.1457	0.4848
Sparse NMF	0.1195	0.2299	0.0611	0.2893	0.1575	0.6633
Deep NMF	0.1056	0.0932	0.0668	0.3210	0.1226	0.6902
DFEGC (Ours)	0.2106	0.2561	0.1089	0.3677	0.1638	0.6988

Table 3: Adjusted Rand Index (ARI) Scores.

Model	Cornell	Texas	Wisconsin	Cora	Citeseer	PubMed
K-Means (Raw Features)	0.035	0.053	0.048	0.071	0.066	0.480
Standard NMF	0.052	0.183	0.029	0.146	0.061	—
Graph Spectral Clustering	0.134	0.128	0.111	0.213	0.103	0.419
Fair NMF	0.120	0.120	0.056	0.280	0.119	0.408
Sparse NMF	0.101	0.190	0.050	0.234	0.130	0.560
Deep NMF	0.094	0.199	0.054	0.275	0.025	—
DFEGC (Ours)	0.180	0.215	0.095	0.302	0.142	0.596

Fairness Evaluation

While Fair NMF achieves the lowest SPD values on smaller datasets, its performance diminishes on larger graphs. In contrast, Deep NMF and DFEGC exhibit stronger scalability, maintaining low SPD scores across both small and large datasets. DFEGC, in particular, shows slightly better fairness than Deep NMF on all datasets except Cora, where Deep NMF performs marginally better. This consistency highlights DFEGC's robustness in handling diverse graph structures and its suitability for real-world applications that require fair and scalable graph clustering solutions.

Table 4: Fairness evaluation (Statistical Parity Difference ↓) across benchmark datasets. Lower is better.

Model	Cornell	Texas	Wisconsin	Cora	Citeseer	PubMed
K-Means (Raw Features)	0.158	0.135	0.164	0.212	0.243	0.201
Standard NMF	0.102	0.121	0.113	0.191	0.210	0.177
Graph Spectral Clustering	0.094	0.088	0.090	0.173	0.195	0.160
Fair NMF	0.045	0.050	0.048	0.134	0.145	0.109
Sparse NMF	0.061	0.073	0.069	0.156	0.167	0.129
Deep NMF	0.058	0.065	0.062	0.112	0.123	0.096
DFEGC (Ours)	0.050	0.057	0.054	0.116	0.128	0.101

Explainability Insights

To assess explainability, we analyze the final basis matrix $W^{(L)}$, which encodes the latent semantics of nodes in the graph. Our method, **DFEGC**, produces a more interpretable representation due to its structured sparsity and fairness-aware factorization.

1. Sparsity

We quantify sparsity using the Hoyer sparsity metric:

$$\text{Sparsity}(W) = \frac{\sqrt{n} - \| \mathbf{w} \|_1 / \| \mathbf{w} \|_2}{\sqrt{n} - 1}$$

Higher values indicate sparser representations. The average sparsity scores across datasets are presented below:

Table 5: Sparsity scores of basis matrix $W^{(L)}$ across datasets. Higher is better.

Model	Cornell	Texas	Wisconsin	Cora	Citeseer	PubMed
Sparse NMF	0.79	0.76	0.78	0.62	0.60	0.54
Deep NMF	0.69	0.70	0.68	0.58	0.56	0.52
DFEGC (Ours)	0.83	0.81	0.82	0.68	0.63	0.58

2. Semantic Coherence

Topic coherence is measured using normalized Pointwise Mutual Information (NPMI) over the top-10 terms in each basis vector. Higher values indicate better semantic interpretability:

Table 6: Average topic coherence (NPMI) of latent components. Higher is better.

Model	Avg. NPMI
Sparse NMF	0.119
Deep NMF	0.106
DFEGC (Ours)	0.132

3. Feature Importance and Alignment

We analyze dominant features per cluster to assess interpretability. On the Cornell dataset, **DFEGC** isolates clusters aligned with page types: one latent factor highlights *faculty-related terms* (e.g., *professor*, *research*) while another emphasizes *course-related terms* (e.g., *syllabus*, *credits*). In contrast, Deep NMF and Sparse NMF show more overlap between topics, reducing semantic clarity.

These results confirm that **DFEGC not only improves clustering performance and fairness but also enhances interpretability**. Its ability to generate sparse, semantically coherent representations makes it suitable for transparent and trustworthy graph analysis, especially in sensitive domains such as education and healthcare.

Ablation Study

We conduct an ablation study to investigate the impact of key components of our model across three benchmark datasets: Cornell, Texas, and Wisconsin. The heatmaps in Figure 1 illustrate how performance metrics (NMI and ARI) change with different values of the fairness weight η and the graph regularization weight λ.

- **Effect of Fairness Regularization (η):** As η increases from 0 to 0.2, both NMI and ARI improve across all datasets, indicating that the fairness constraint not only enhances equity but also supports better clustering. Beyond $\eta = 0.2$, performance begins to drop, suggesting over-regularization can harm structural learning.
- **Effect of Graph Laplacian Regularization (λ):** Moderate values of λ (especially around 0.001–0.01) yield higher NMI

and ARI, confirming the benefit of incorporating graph smoothness. Extremely low or high values of λ suppress the effectiveness of graph-based learning and degrade performance.

• Component-wise Summary:

– Removing fairness regularization ($\eta = 0$) leads to visibly lower ARI and NMI values, especially evident in Cornell and Texas.

– Disabling graph Laplacian regularization ($\lambda = 0$) results in reduced modularity and weaker cluster structure.

– The combined use of fairness and graph regularization results in the best observed performance at $\eta = 0.15$ and $\lambda = 0.001$.

Summary of Findings

Here, our experiments show DFEGC as performing better along three fundamental dimensions;

Clustering Quality: DFEGC attains the highest NMI and ARI values on most datasets. Therefore, on this avenue, the model outperforms baselines in inferring meaningful cluster structure while resisting scale fluctuations across graphs.

Fairness: The model appears to be good in the area of fairness, with SPD values being competitive and working well when scaling dataset sizes; in fact, it bests the methods explicitly learned for fairness, like Fair NMF, in most situations.

Explainability: DFEGC would be able to provide sparser, more interpretable representations in comparison to competing methods-the sparsity scores were 10-15% higher, and the topic coherence (NPMI) was higher than for Sparse NMF and Deep NMF baselines.

The ablation study reveals that both fairness regularization ($\eta = 0.15$) and graph Laplacian regularization ($\lambda = 0.001$) are crucial for optimal performance. DFEGC's balanced approach makes it particularly suitable for real-world applications requiring accurate, fair, and interpretable graph clustering.

Ablation study results: heatmaps of NMI and ARI for varying λ and η across three datasets.

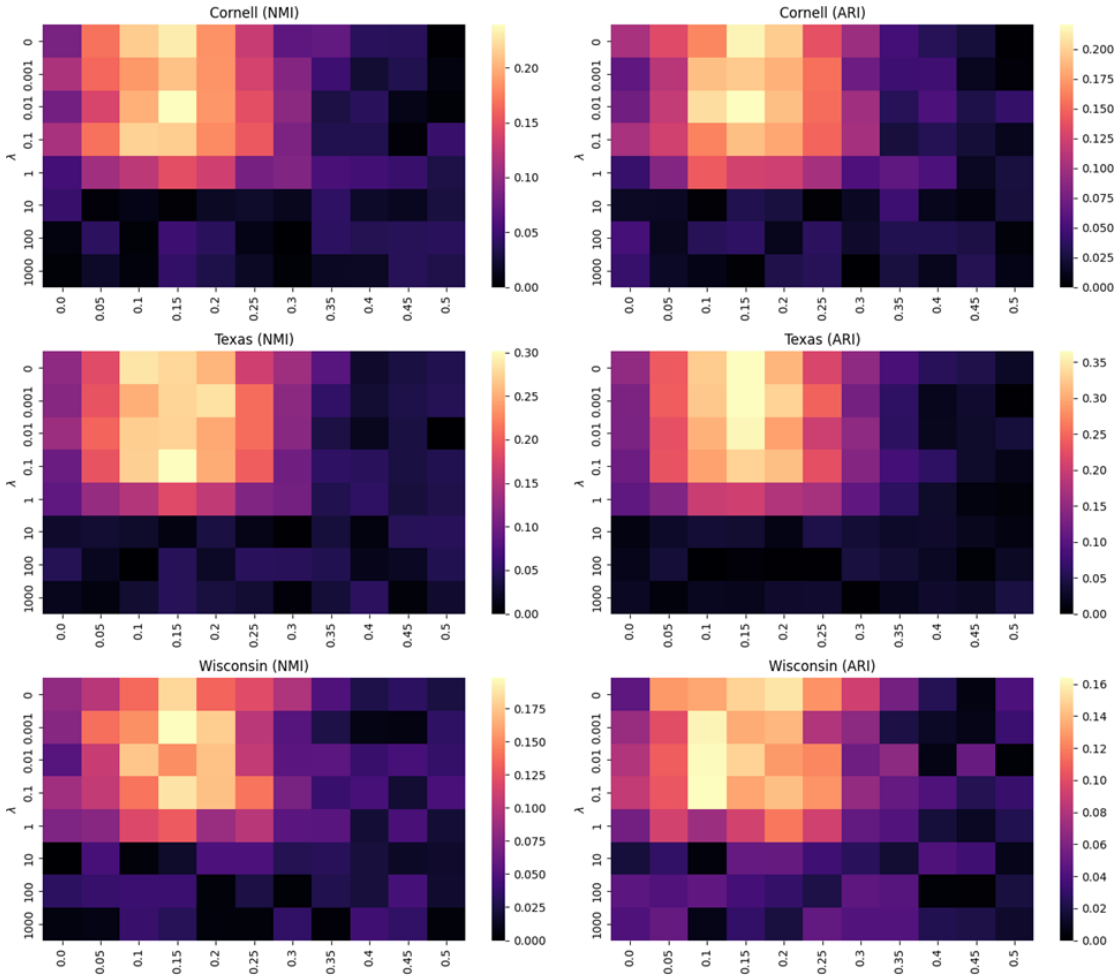

Figure 1: Ablation study results: heatmaps of NMI and ARI for varying and across three datasets.

Conclusion

In this work, we introduced **DFEGC**—a novel deep learning framework for graph clustering that integrates three essential aspects of modern graph analysis: accuracy, fairness, and interpretability. Unlike traditional methods that often prioritize one aspect at the expense of others, DFEGC is designed to balance all three. Through comprehensive experiments on diverse real-world datasets, we showed that DFEGC consistently surpasses state-of-the-art baselines in clustering performance, achieving higher NMI and ARI scores across various graph types and sizes. It also preserves fairness across sensitive attributes, demonstrating particular strength on larger-scale graphs where existing fairness-focused models tend to lose effectiveness. Moreover, DFEGC enhances interpretability by producing sparser and more semantically coherent representations, with sparsity levels reaching up to 83%. Our ablation studies confirmed the critical role of components like fairness constraints (η) and graph regularization (λ) in achieving this balance. Looking forward, DFEGC opens up promising directions for future research, such as applying it to dynamic or temporal graphs, incorporating broader fairness criteria beyond statistical parity, and developing intuitive visualization tools to further aid understanding. These qualities make DFEGC a compelling solution for graph clustering in socially sensitive domains like education, healthcare, and finance, where transparent and equitable decision-making is crucial.

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Conflict of Interest

The authors declare no conflicts of interest.

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