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Original Article

Cubic Q-positive implicative ideals in Q-algebras

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ABSTRACT

A cubic Q-algebra is a generalization of a fuzzy Q-algebra. This work explores important concepts in Q-algebras, including cubic Q-positive implicative ideals, cubic Q-implicative ideals, and cubic Q-commutative ideals, and investigates their fundamental properties. The most important results obtained are: In a commutative Q-algebra. If any ideal is a cubic positive Q-implicative ideal, then it is a cubic Q-implicative ideal. But the converse does not hold. There is no connection between cubic positive Q-implicative ideals and cubic Q-commutative ideals, nor is there a relationship between cubic Q-implicative ideals and cubic Q-commutative ideals. Any cubic Q-positive implicative ideal is a cubic BCK-ideal, and the converse statement is true under certain conditions.

Keywords: Q-algebra, cubic Q-ideal, cubic Q-positive implicative ideals, cubic Q-commutative ideals, a cubic Q-implicative ideal.

1 Introduction

Two new classes of abstract algebras, BCK and BCI-algebra, were introduced by Imai and Iseki [1]. BCI-algebra is a generalization of BCK-algebra. Many papers presented this structure in different ways; see [2, 3, 4, and 5].

Hu and Li [6] studied BCH-algebra, and they showed that the class of BCI-algebra is a proper subclass of BCH-algebra. Neggers et al. [7] introduced the notion of Q-algebra, which is a generalization of BCK/BCI/BCH-algebras. In 2009, Abdel Naby et al. [8] introduced the fuzzy Q-ideal in Q-algebra, and Mostafa et al. [9 and 10] studied the notions of interval-valued fuzzy Q-ideals and intuitionistic fuzzy Q-ideals. After that, Abdullah and Jawad [11] discussed new types of Pseudo ideals in Pseudo Q-algebra.

Combining the idea of fuzzy sets and interval-valued fuzzy sets, Jun et al. [12 and 13] introduced the concept of cubic sets. They applied it to a subalgebra and a q-ideal of a BCK-algebra and studied some of the relations between them. Yaqoob et al [14] studied a cubic KU-ideals and they proved some important results. Some papers introduced a cubic set in different ways, see [15, 16 and 17].

Satyanarayana and Prasad [18] presented some results on fuzzy (implicative, positive implicative and commutative) ideals in BCK-algebras. Senapati and Shum[19] studied cubic implicative ideals for a BCK-algebra and kareem et al. [20] studied the concept of cubic positive implicative ideals in a KU-semigroup. They study a cubic positive implicative ideal and the relation between a cubic positive implicative ideal and cubic implicative (commutative) ideals. Also, these types of ideals can be applied to several topological ideals and open sets in topological spaces, see [21, 22, 23, 24, 25 and 26].

In this work, the three ideals in Q-algebra, such as cubic (Q-positive implicative, Q-implicative, and Q-commutative) ideals, are studied, and some relationship among these types of ideals is discussed.

2 Basic concepts

Below, we will review some useful definitions, examples, and theories to complete our work.

Definition 2. 1[1]: A BCK-algebra defined as a non-empty set $\aleph$ with a binary relation $*$ and a constant 0 satisfying: For all $\theta, ƛ, \epsilon \in \aleph$

BCI_1 : $[(\theta * ƛ) * (\theta * \epsilon)] * (\epsilon * ƛ) = 0$,

BCI_2 : $(\theta * (\theta * ƛ)) * ƛ = 0$,

BCI_3 : $\theta * \theta = 0$,

BCI_4 : $\theta * ƛ = 0$ and $ƛ * \theta = 0$ implies $\theta = ƛ$,

BCI_5 : $0 * \theta = 0$.

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In a BCK-algebra $(\aleph, * ,0)$, a binary operation $\leq$ define by $\theta \leq \lambda$ if and only if $\theta * \lambda = 0$.

And then the following axioms hold. For all $\theta, \lambda, \epsilon \in \aleph$,

BCI$'_1$: $[(\theta * \lambda) * (\theta * \epsilon)] \leq (\epsilon * \lambda)$,

BCI$'_2$: $(\theta * (\theta * \lambda)) \leq \lambda$,

BCI$'_3$: $\theta \leq \theta$,

BCI$'_4$: $\theta \leq \lambda$ and $\lambda \leq \theta$ implies $\theta = \lambda$ and

BCI$'_5$: $0 \leq \theta$.

Proposition2. 2 [1]: In a BCK-algebra $\aleph$, we have $(\theta * \lambda) * \epsilon = (\theta * \epsilon) * \lambda, \forall \theta, \lambda, \epsilon \in \aleph$.

Definition2. 3 [1]: In a BCK-algebra $(\aleph, * ,0)$. The non-empty subset ψ of $\aleph$ is called a BCK-ideal iff

1- $0 \in \psi$

2- $\theta * \lambda \in I$ and $\lambda \in \psi$ imply $\theta \in \psi$.

Definition2. 4 [7]: A Q-algebra $(\aleph, * ,0)$ is a nonempty subset $\aleph$ with a constant "0" and a binary operation " $*$" satisfying the following:

(Q1) $\theta * \theta = 0$

(Q2) $\theta * 0 = \theta$,

(Q3) $(\theta * \lambda) * \epsilon = (\theta * \epsilon) * \lambda, \forall \theta, \lambda, \epsilon \in \aleph$.

In a Q-algebra $(\aleph, * ,0)$, a binary operation $\leq$ define by $\theta \leq \lambda$ if and only if $\theta * \lambda = 0$.

Example 2. 5 [7]: Let $\aleph$={0, x, z }be a set as shown in the table below.

Table 1: A Q-algebra.

$*$	**0**	**z**	$\boldsymbol{x}$
0	0	0	0
z	z	0	0
$\boldsymbol{x}$	x	0	0

Then $(\aleph, * ,0)$ is a Q-algebra.

Proposition 2. 6 [7]: If $(\aleph, * ,0)$ is a Q-algebra, then $\left(\theta * (\theta * \lambda)\right) * \lambda = 0$ for any $\theta, \lambda \in \aleph$.

Definition2. 7: A Q-algebra $\aleph$ is said to be commutative if it satisfies the identity

$\left(\theta * (\theta * \lambda)\right) = \left(\lambda * (\lambda * \theta)\right)$, for all $\theta, \lambda \in \aleph$.

Example 2. 8: Let $\aleph = \{0, l, m, n\}$ be a set and $*$ a binary operation defined as follows

Table 2: a commutative Q-algebra.

$*$	0	$\boldsymbol{l}$	$\boldsymbol{m}$	$\boldsymbol{n}$
0	0	0	0	0
$\boldsymbol{l}$	l	0	0	l
$\boldsymbol{m}$	m	l	0	m
$\boldsymbol{n}$	n	n	n	0

It is easy to show that $\aleph$ is a commutative Q-algebra.

Definition 2. 9 [7]:. In a Q-algebra $(\aleph, * ,0)$. The non-empty subset $\mathcal{L}$ of $\aleph$ is called a Q-subalgebra if $\theta * \lambda \in \mathcal{L}$ whenever $\theta, \lambda \in \mathcal{L}$.

Definition 2.10 [7]:. In a Q-algebra $(\aleph, * ,0)$. The non-empty subset ψ of $\aleph$ is called a Q-ideal if,

i) $0 \in \psi$,

ii) $\forall \theta, \lambda, \epsilon \in \aleph, ((\theta * \lambda) * \epsilon)) \in \psi,\ \lambda \in \psi$ imply $\theta * \epsilon \in \psi$.

Example2. 1 1 [7]: Let $\aleph = \{0, l , m , n, k\}$ be a set defined by the table below.

Table 3: A Q-ideal.

$*$	0	$\boldsymbol{l}$	$\boldsymbol{m}$	$\boldsymbol{n}$	$\boldsymbol{k}$
0	0	0	0	0	k
$\boldsymbol{l}$	l	0	0	l	k
$\boldsymbol{m}$	m	m	0	0	k
$\boldsymbol{n}$	n	0	n	0	k
$\boldsymbol{k}$	k	k	k	k	0

Then $(\aleph, * ,0)$ is a Q-algebra and it is clear that the set $\psi = \{0, l, m, n\}$ is a Q-ideal of $\aleph$.

Proposition 2.12 [7]: In a Q-algebra $(\aleph, * ,0)$. Every Q-ideal of $\aleph$ is a BCK-ideal.

Definition 2.13: In a Q-algebra $(\aleph, * ,0)$. The non-empty subset ψ of $\aleph$ is called a commutative Q-ideal if, for each $\theta, \lambda, \epsilon \in \aleph$,

i) $0 \in \psi$,

ii) $((\theta * \lambda) * \epsilon)) \in \psi, \epsilon \in \psi$ imply $(\theta * (\lambda * (\lambda * \theta) \in \psi$.

Definition 2.14 [3]: In a Q-algebra $(\aleph, * ,0)$. A fuzzy set μ in $\aleph$ is called a fuzzy BCK-ideal of $\aleph$, if

(1) $\mu(0) \geq \mu(\theta)$, for any $\theta \in \aleph$

(2) $\mu(\theta) \geq \min\{\mu(\theta * \lambda), \mu(\lambda)\}$, for any θ, λ in $\aleph$.

Definition 2.15 [8]: In a Q-algebra $(\aleph, * ,0)$. A fuzzy set μ in $\aleph$ is called a fuzzy Q-ideal, if

(1) $\mu(0) \geq \mu(\theta)$, for any $\theta \in \aleph$

(2) $\mu(\theta * \epsilon) \geq \min\{\mu((\theta * ℷ) * \epsilon), \mu(ℷ)\}$, for any $\theta, ℷ, \epsilon$ in $\aleph$.

Definition 2. 16 [27]: An interval valued fuzzy set is $V =< \theta, \tilde{\pi}(\theta) >$, where $\tilde{\pi}: \aleph \to D[0,1]$, defined as $\tilde{\pi} = \{(\theta, [\pi^L(\theta), \pi^U(\theta)]): \theta \in \aleph\}$, where π^L and π^U are two fuzzy sets such that $\pi^L(\theta) \leq \pi^U(\theta)$ and $D[0,1] = \{[m^L, m^U]: m^L \leq m^U \ for\ m^L, m^U \in [0,1]\}$, which is all closed sub-intervals of [0,1].

Definition 2.17 [13]: The structure $\varpi = \{\langle \theta, \tilde{\pi}_\varpi(\theta), \alpha_\varpi(\theta) \rangle : \theta \in \aleph\}$ is called a cubic set in $\aleph$, such that

$\tilde{\pi}_\varpi(\theta) = [\pi^L_\varpi(\theta), \pi^U_\varpi(\theta)]$, where $\tilde{\pi}_\varpi : \aleph \to D[0,1]$ is an interval valued fuzzy set and $\alpha_\varpi: \aleph \to [0,1]$ is a fuzzy set. The cubic set can be written in short form as $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$. The level subset of $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ which is denoted by $U(\varpi, \tilde{t}, r)$ can be defined as $U(\varpi, \tilde{t}, r) = \{\theta \in \aleph: \tilde{\pi}_\varpi(\theta) \succcurlyeq \tilde{t}, \alpha_\varpi \leq r\}$, for every $[0,0] \preccurlyeq \tilde{t} \preccurlyeq [1,1]$ and $r \in [0,1]$.

Definition 2.18 [28]: In a Q-algebra$\aleph$. A cubic set $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is called a cubic Q-sub-algebra if for each $\theta, ℷ \in \aleph$. Then

(q_1) $\tilde{\pi}_\varpi(\theta * ℷ) \succcurlyeq rmin\{\tilde{\pi}_\varpi(\theta), \tilde{\pi}_\varpi(ℷ)\}$.

(q_2) $\alpha_\varpi(\theta * ℷ) \leq max\{\alpha_\varpi(\theta), \alpha_\varpi(ℷ)\}, \forall \theta, ℷ \in \aleph$.

Definition 2.19 [12]: In a Q-algebra$\aleph$. A cubic set $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is called a cubic BCK-ideal if: for each $\theta, ℷ \in \aleph$.

(H_1)$\tilde{\pi}_\varpi(0) \succcurlyeq \tilde{\pi}_\varpi(\theta)$ and $\alpha_\varpi(0) \leq \alpha_\varpi(\theta)$.

(H_2) $\tilde{\pi}_\varpi(\theta) \succcurlyeq rmin\{\tilde{\pi}_\varpi(\theta * ℷ), \tilde{\pi}_\varpi(ℷ)\}$ and $\alpha_\varpi(\theta) \leq max\{\alpha_\varpi(\theta * ℷ), \alpha_\varpi(ℷ)\}$.

Definition 2. 20 [28]: In a Q-algebra$\aleph$. A cubic set $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is called a cubic Q-ideal if: for each $\theta, ℷ, \epsilon \in \aleph$.

$(B_1)\ \tilde{\pi}_\varpi(0) \succcurlyeq \tilde{\pi}_\varpi(\theta)\ and\ \alpha_\varpi(0) \leq \alpha_\varpi(\theta),$

$(B_2)\ \tilde{\pi}_\varpi(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ℷ) * \epsilon), \tilde{\pi}_\varpi(ℷ)\}\ and$

$\alpha_\varpi(\theta * \epsilon) \leq max\{\alpha_\varpi((\theta * ℷ) * \epsilon), \alpha_\varpi(ℷ)\}.$

3 Cubic Q-positive implicative-ideals of a Q-algebra

This part presents definitions of the cubic Q-positive implicative ideal, the cubic Q-implicative ideal, and the cubic Q-commutative ideal of Q-algebras, as well as some of the relationships that connect these ideals.

Definition 3. 1: In a Q-algebra$\aleph$. The cubic set $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is called **a cubic Q-positive implicative ideal** if for each $\theta, ℷ, \epsilon \in \aleph$

$(\boldsymbol{Cp_1})$ $\tilde{\pi}_\varpi(0) \succcurlyeq \tilde{\pi}_\varpi(\theta)$, and $\alpha_\varpi(0) \leq \alpha_\varpi(\theta)$

$(\boldsymbol{Cp_2})$ $\tilde{\pi}_\varpi(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ℷ) * \epsilon), \tilde{\pi}_\varpi(ℷ * \epsilon)\},$

$\alpha_\varpi(\theta * \epsilon) \leq max\{\alpha_\varpi((\theta * ℷ) * \epsilon), \alpha_\varpi(ℷ * \epsilon)\}.$

Example 3.2: Let $\aleph = \{0, l, m, n, k\}$ be a set defined as example 2.11 and define $\tilde{\pi}_\varpi(\theta)$ and $\alpha_\varpi(\theta)$ by

Table 4: a commutative Q-algebra.

$\aleph$	$\tilde{\pi}_\varpi(\theta)$	$\alpha_\varpi(\theta)$
0	[0.1,0.9]	0.1
$\boldsymbol{l}$	[0.1,0.7]	0.2
$\boldsymbol{m}$	[0.1,0.6]	0.4
$\boldsymbol{n}$	[0.1,0.3]	0.3
$\boldsymbol{k}$	[0.1,0.3]	0.5

We can simply prove that $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic Q-positive implicative ideal**.**

Proposition 3. 3: In a Q-algebra$\aleph$. Let $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ be a cubic Q-positive implicative ideal and $\theta \leq ℷ$, then $\tilde{\pi}_\varpi(\theta) \succcurlyeq \tilde{\pi}_\varpi(ℷ)$ and $\alpha_\varpi(ℷ) \leq \alpha_\varpi(\theta)$.

Proof.

Since $\theta \leq ℷ \Rightarrow \theta * ℷ = 0$, since $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic Q-positive implicative ideal, then

$\tilde{\pi}_\varpi(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ℷ) * \epsilon), \tilde{\pi}_\varpi(ℷ * \epsilon)\}$, put $\epsilon = 0$

$\tilde{\pi}_\varpi(\theta * 0) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ℷ) * 0), \tilde{\pi}_\varpi(ℷ * 0)\}$,
$\tilde{\pi}_\varpi(\theta) \succcurlyeq rmin\{\tilde{\pi}_\varpi(\theta * ℷ), \tilde{\pi}_\varpi(ℷ)\} = rmin\{\tilde{\pi}_\varpi(0), \tilde{\pi}_\varpi(ℷ)\} = \tilde{\pi}_\varpi(ℷ)$, also

$\alpha_\varpi(\theta * \epsilon) \leq max\{\alpha_\varpi((\theta * ℷ) * \epsilon), \alpha_\varpi(ℷ * \epsilon)\}$, put $\epsilon = 0$

$\alpha_\varpi(\theta * 0) \leq max\{\alpha_\varpi((\theta * ℷ) * 0), \alpha_\varpi(ℷ * 0)\}$

$\alpha_\varpi(\theta) \leq max\{\alpha_\varpi(\theta * ℷ), \alpha_\varpi(ℷ)\} = max\{\alpha_\varpi(0), \alpha_\varpi(ℷ)\} = \alpha_\varpi(ℷ)$.□

Proposition 3. 4: Let $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ be a cubic Q-positive implicative ideal. If the inequality $\theta * ℷ \leq \epsilon$ holds, then $\tilde{\pi}_\varpi(\theta) \succcurlyeq rmin\{\tilde{\pi}_\varpi(\epsilon), \tilde{\pi}_\varpi(ℷ)\}$ and $\alpha_\varpi(\theta) \leq max\{\alpha_\varpi(\epsilon), \alpha_\varpi(ℷ)\}$.

Proof. Let $\theta * ℷ \leq \epsilon$, then by pro. 3.3. $\tilde{\pi}_\varpi(\theta * ℷ) \succcurlyeq \tilde{\pi}_\varpi(\epsilon)$ and $\alpha_\varpi(\theta * ℷ) \leq \alpha_\varpi(\epsilon)$ and since $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic Q-positive implicative ideal, then

$\tilde{\pi}_\varpi(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ℷ) * \epsilon), \tilde{\pi}_\varpi(ℷ * \epsilon)\}$, put $\epsilon = 0$

$\tilde{\pi}_\varpi(\theta * 0) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ℷ) * 0), \tilde{\pi}_\varpi(ℷ * 0)\}$
$\tilde{\pi}_\varpi(\theta) \succcurlyeq rmin\{\tilde{\pi}_\varpi(\theta * ℷ), \tilde{\pi}_\varpi(ℷ)\}$, but $\tilde{\pi}_\varpi(\theta * ℷ) \succcurlyeq \tilde{\pi}_\varpi(\epsilon)$, then

$\tilde{\pi}_\varpi(\theta) \succcurlyeq rmin\{\tilde{\pi}_\varpi(\epsilon), \tilde{\pi}_\varpi(ℷ)\}$, also

$\alpha_\varpi(\theta * \epsilon) \leq max\{\alpha_\varpi((\theta * ℷ) * \epsilon), \alpha_\varpi(ℷ * \epsilon)\}$, put $\epsilon = 0$

$\alpha_\varpi(\theta * 0) \leq max\{\alpha_\varpi((\theta * ℷ) * 0), \alpha_\varpi(ℷ * 0)\}$

$\alpha_\varpi(\theta) \leq max\{\alpha_\varpi(\theta * ℷ), \alpha_\varpi(ℷ)\}$, but $\alpha_\varpi(\theta * ℷ) \leq \alpha_\varpi(\epsilon)$, then

$\alpha_\varpi(\theta) \leq max\{\alpha_\varpi(\epsilon), \alpha_\varpi(ℷ)\}$, That's what's needed. □

Theorem 3. 5: Any a cubic Q-positive implicative ideal of $\aleph$ is a cubic BCK-ideal.

Proof. Let $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ be a cubic Q-positive implicative ideal, then

$\tilde{\pi}_\varpi(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ג) * \epsilon), \tilde{\pi}_\varpi(ג * \epsilon)\}$,

$\alpha_\varpi(\theta * \epsilon) \le max\{\alpha_\varpi((\theta * ג) * \epsilon), \alpha_\varpi(ג * \epsilon)\}$, put $\epsilon = 0$ we get

$\tilde{\pi}_\varpi(\theta * 0) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ג) * 0), \tilde{\pi}_\varpi(ג * 0)\}$,

$\tilde{\pi}_\varpi(\theta) \succcurlyeq rmin\{\tilde{\pi}_\varpi(\theta * ג), \tilde{\pi}_\varpi(ג)\}$ … … . . (1) , and

$\alpha_\varpi(\theta * 0) \le max\{\alpha_\varpi((\theta * ג) * 0), \alpha_\varpi(ג * 0)\}$

$\alpha_\varpi(\theta) \le max\{\alpha_\varpi(\theta * ג), \alpha_\varpi(ג)\}$ … … … (2)

By (1) and (2) implying that $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic BCK-ideal.

The next example indicates that the converse of the above theorem does not hold.

Example3. 6: Let $\aleph = \{0, l, m, n\}$ be a set defined by the table below.

Table 5: a Q-algebra.

$*$	0	l	m	n
0	0	0	0	0
l	l	0	0	l
m	m	m	0	m
n	n	n	n	0

The triple $(\aleph, *, 0)$ is a Q-algebra. Define $\tilde{\pi}_\varpi(\theta)$ and $\alpha_\varpi(\theta)$ by

Table 6: a cubic BCK-ideal.

$\aleph$	$\tilde{\pi}_\varpi(\theta)$	$\alpha_\varpi(\theta)$
0	[0.1,0.8]	0.1
l	[0.1,0.2]	0.5
m	[0.1,0.4]	0.4
n	[0.1,0.5]	0.4

We can simply prove that $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic BCK-ideal but it is not a cubic Q-positive implicative ideal, since

$\tilde{\pi}_\varpi(l * n) = [0.1,0.2] \prec rmin\{\tilde{\pi}_\varpi((l * m) * n), \tilde{\pi}_\varpi(m * n)\}$,

$\alpha_\varpi(l * n) > max\{\alpha_\varpi((l * m) * n), \alpha_\varpi(m * n)\}$.

Theorem 3. 7: Let $(\aleph, *, 0)$ be a Q-algebra. If $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic Q-positive implicative ideal of $\aleph$, then $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic BCK-ideal and satisfying the condition

$\tilde{\pi}_\varpi(\theta * ג) \succcurlyeq \tilde{\pi}_\varpi((\theta * ג) * ג))$, $\alpha_\varpi(\theta * ג) \le \alpha_\varpi((\theta * ג) * ג)), \forall \theta, ג \in \aleph$.

Proof. Let $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ be a cubic Q-positive implicative ideal, then $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic BCK-ideal and taking $\epsilon = ג$ in definition 3.1. It follows that

$$\tilde{\pi}_\varpi(\theta * ג) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ג) * ג), \tilde{\pi}_\varpi(ג * ג)\}$$

$$= rmin\{\tilde{\pi}_\varpi((\theta * ג) * ג), \tilde{\pi}_\varpi(0)\}$$

$$= \tilde{\pi}_\varpi((\theta * ג) * ג)),$$ and

$$\alpha_\varpi(\theta * ג) \le \max\{\alpha_\varpi((\theta * ג) * ג), \alpha_\varpi(ג * ג)\}$$

$$= \max\{\alpha_\varpi((\theta * ג) * ג), \alpha_\varpi(0)\}$$

$$= \alpha_\varpi((\theta * ג) * ג)). \square$$

Now, the following theorem explains the relationship between the cubic BCK ideal and the cubic Q-positive implicative ideal under certain conditions.

Theorem 3. 8: Let $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ be a cubic BCK-ideal, then $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic Q-positive implicative ideal $\Leftrightarrow$ The following condition holds, for each $\theta, ג \in \aleph$.

$\tilde{\pi}_\varpi((\theta * \epsilon) * (ג * \epsilon)) \succcurlyeq \tilde{\pi}_\varpi((\theta * ג) * \epsilon)$, $\alpha_\varpi((\theta * \epsilon) * (ג * \epsilon)) \le \alpha_\varpi((\theta * ג) * \epsilon)$.

Proof. Let $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ be a cubic BCK-ideal satisfying the above condition, we have $\forall \theta, ג \in \aleph$, $\tilde{\pi}_\varpi(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * \epsilon) * (ג * \epsilon)), \tilde{\pi}_\varpi(ג * \epsilon)\}$

$$\succcurlyeq rmin\{\tilde{\pi}_\varpi((\theta * ג) * \epsilon)), \tilde{\pi}_\varpi(ג * \epsilon)\}$$

and $\alpha_\varpi(\theta * \epsilon) \le max\{\alpha_\varpi((\theta * \epsilon) * (ג * \epsilon)), \alpha_\varpi(ג * \epsilon)\}$.

$$\le max\{\alpha_\varpi((\theta * ג) * \epsilon)), \alpha_\varpi(ג * \epsilon)\}.$$

Hence $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ is a cubic Q-positive implicative ideal.

Conversely, let $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ be a cubic Q-positive implicative ideal and it is a cubic BCK-ideal.

Now, let $m = (\theta * (ג * \epsilon))$ *and* $n = \theta * ג$, then

$\tilde{\pi}_\varpi((m * n) * \epsilon) = \tilde{\pi}_\varpi([(\theta * (ג * \epsilon)) * (\theta * ג)] * \epsilon)$ and by BCI_1', we have

$\tilde{\pi}_\varpi([(\theta * (ג * \epsilon)) * (\theta * ג)] * \epsilon) \preccurlyeq \tilde{\pi}_\varpi((ג * (ג * \epsilon)) * \epsilon)$ and by Proposition2.2, we get

$\tilde{\pi}_\varpi((ג * (ג * \epsilon)) * \epsilon) = \tilde{\pi}_\varpi((ג * \epsilon) * (ג * \epsilon)) = \tilde{\pi}_\varpi(0)$. Also,

$\alpha_\varpi((m * n) * \epsilon) = \alpha_\varpi((\theta * (ג * \epsilon)) * (\theta * ג)) * \epsilon)$

$\le \alpha_\varpi((ג * (ג * \epsilon)) * \epsilon) = \alpha_\varpi((ג * \epsilon) * (ג * \epsilon)) = \alpha_\varpi(0)$

It follows that, by applied proposition2.2 and the condition $\boldsymbol{Cp_2}$ we obtain

$\tilde{\pi}_\varpi((\theta * \epsilon) * (ג * \epsilon)) = \tilde{\pi}_\varpi((\theta * (ג * \epsilon)) * \epsilon)$

$= \tilde{\pi}_{\varpi}(m * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((m * n) * \epsilon)), \tilde{\pi}_{\varpi}(n * \epsilon)\}$

$= rmin\{\tilde{\pi}_{\varpi}(0), \tilde{\pi}_{\varpi}((\theta * \lambda) * \epsilon)\} = \tilde{\pi}_{\varpi}((\theta * \lambda) * \epsilon)$. Also,

$\alpha_{\varpi}((\theta * \epsilon) * (\lambda * \epsilon)) = \alpha_{\varpi}((\theta * (\lambda * \epsilon)) * \epsilon)$

$= \alpha_{\varpi}(m * \epsilon)$

$\leq max\{\alpha_{\varpi}((m * n) * \epsilon)), \alpha_{\varpi}(n * \epsilon)\}$

$= max\{\alpha_{\varpi}(0), \alpha_{\varpi}((\theta * \lambda) * \epsilon)\} = \alpha_{\varpi}((\theta * \lambda) * \epsilon)$. □

Definition 3.9. In a Q-algebra$(\aleph, *, 0)$. The cubic set $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ of $\aleph$ is named **a cubic Q-implicative ideal** if $\forall\ \theta, \lambda, \epsilon \in \aleph$

$(\boldsymbol{Ci_1})$ $\tilde{\pi}_{\varpi}(0)) \succcurlyeq \tilde{\pi}_{\varpi}(\theta)$ and $\alpha_{\varpi}(0) \leq \alpha_{\varpi}(\theta)$

$(\mathbf{Ci_2})$ $\tilde{\pi}_{\varpi}(\theta * (\lambda * \theta)) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}(\theta * (\lambda * \theta)) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\}$,

$\alpha_{\varpi}(\theta * (\lambda * \theta)) \leq max\{\alpha_{\varpi}(\theta * (\lambda * \theta)) * \epsilon), \alpha_{\varpi}(\epsilon)\}$.

Theorem 3. 10: The cubic BCK-ideal of $\aleph$ is a cubic Q-implicative ideal if it satisfies the condition

$\tilde{\pi}_{\varpi}(\theta) \succcurlyeq \tilde{\pi}_{\varpi}(\theta * (\lambda * \theta))$ and $\alpha_{\varpi}(\theta) \leq \alpha_{\varpi}(\theta * (\lambda * \theta))$, $\forall\ \theta, \lambda, \epsilon \in \aleph$

Proof. Suppose $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ is a cubic BCK-ideal and satisfies the above condition, it follows that

$\tilde{\pi}_{\varpi}(\theta) \succcurlyeq \tilde{\pi}_{\varpi}(\theta * (\lambda * \theta)) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}(\theta * (\lambda * \theta)) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\}$ and

$\alpha_{\varpi}(\theta) \leq \alpha_{\varpi}(\theta * (\lambda * \theta)) \leq max\{\alpha_{\varpi}(\theta * (\lambda * \theta)) * \epsilon), \alpha_{\varpi}(\epsilon)\}$. Then

$\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ is a cubic Q-implicative ideal of $\aleph$. □

Theorem 3.11: In a commutative Q-algebra $\aleph$. If $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ of $\aleph$ is a cubic positive Q-implicative ideal, then it is a cubic Q-implicative ideal.

Proof. Since $\aleph$ is a commutative Q-algebra and by using Theorem 3.8 and (Q_3), we have

$\tilde{\pi}_{\varpi}(\theta * (\theta * (\lambda * \theta))) = \tilde{\pi}_{\varpi}((\lambda * \theta) * ((\lambda * \theta) * \theta)) \succcurlyeq \tilde{\pi}_{\varpi}\left((\lambda * (\lambda * \theta)) * \theta\right)$

$= \tilde{\pi}_{\varpi}(\lambda * \theta) * (\lambda * \theta) = \tilde{\pi}_{\varpi}(0)$. Also,

$\alpha_{\varpi}(\theta * (\theta * (\lambda * \theta))) = \alpha_{\varpi}((\lambda * \theta) * ((\lambda * \theta) * \theta))$
$\leq \alpha_{\varpi}\left((\lambda * (\lambda * \theta)) * \theta\right)$

$= \alpha_{\varpi}(\lambda * \theta) * (\lambda * \theta) = \alpha_{\varpi}(0)$

By Theorem 3.5, we have $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ is a cubic BCK-ideal, then

$\tilde{\pi}_{\varpi}(\theta) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * (\theta * (\lambda * \theta))), \tilde{\pi}_{\varpi}(\theta * (\lambda * \theta))\}$

$\succcurlyeq rmin\{\tilde{\pi}_{\varpi}(0), \tilde{\pi}_{\varpi}(\theta * (\lambda * \theta))\} = \tilde{\pi}_{\varpi}(\theta * (\lambda * \theta))$. Also,

$\alpha_{\varpi}(\theta) \leq max\{\alpha_{\varpi}((\theta * (\theta * (\lambda * \theta))), \alpha_{\varpi}(\theta * (\lambda * \theta))\}$

$= max\{\alpha_{\varpi}(0), \alpha_{\varpi}(\theta * (\lambda * \theta))\} = \alpha_{\varpi}(\theta * (\lambda * \theta))$. Therefore from theorem 3.10, we get $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ is a cubic Q-implicative ideal. □

Definition 3. 12: In a Q-algebra$(\aleph, *, 0)$. The cubic set $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ of $\aleph$ is named **a cubic Q-commutative ideal** if $\forall\ \theta, \lambda, \epsilon \in \aleph$

$(\boldsymbol{CC_1})$ $\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{\pi}_{\varpi}(\theta)$, and $\alpha_{\varpi}(0) \leq \alpha_{\varpi}(\theta)$

$(\boldsymbol{CC_2})$ $\tilde{\pi}_{\varpi}(\theta * (\lambda * (\lambda * \theta))) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \lambda) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\}$,

$\alpha_{\varpi}(\theta * (\lambda * (\lambda * \theta))) \leq max\{\alpha_{\varpi}((\theta * \lambda) * \epsilon), \alpha_{\varpi}(\epsilon)\}$.

Example 3.13: Let $\aleph = \{0, l, m, n\}$ be a set an example 3.6. We can define $\tilde{\pi}_{\varpi}(\theta)$ and $\alpha_{\varpi}(\theta)$ by

Table 7: a cubic Q-commutative ideal.

$\aleph$	$\tilde{\pi}_{\varpi}(\theta)$	$\alpha_{\varpi}(\theta)$
0	[0.5,0.8]	0.2
$\boldsymbol{l}$	[0.4,0.5]	0.3
$\boldsymbol{m}$	[0.1,0.3]	0.5
$\boldsymbol{n}$	[0.1,0.3]	0.5

Then it is easy to prove that $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ is a cubic Q-commutative ideal**.**

Theorem 3.14: In Q-algebra $\aleph$. The cubic Q-ideal of $\aleph$ is a cubic Q-commutative ideal if and only if it satisfies the two conditions, for each $\theta, \lambda \in \aleph$.

(a) $\tilde{\pi}_{\varpi}(\theta * \lambda) \preccurlyeq \tilde{\pi}_{\varpi}(\theta * (\lambda * (\lambda * \theta)))$ and **(b)** $\alpha_{\varpi}(\theta * \lambda) \geq \alpha_{\varpi}(\theta * (\lambda * (\lambda * \theta)))$.

Proof. Let $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ be a cubic Q-commutative ideal, then

$\tilde{\pi}_{\varpi}(\theta * (\lambda * (\lambda * \theta))) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \lambda) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\}$, and

$\alpha_{\varpi}(\theta * (\lambda * (\lambda * \theta))) \leq max\{\alpha_{\varpi}((\theta * \lambda) * \epsilon), \alpha_{\varpi}(\epsilon)\}$. Taking $\epsilon = 0$, we get

$\tilde{\pi}_{\varpi}(\theta * (\lambda * (\lambda * \theta))) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \lambda) * 0), \tilde{\pi}_{\varpi}(0)\} = \tilde{\pi}_{\varpi}(\theta * \lambda)$. Also,

$\alpha_{\varpi}(\theta * (\lambda * (\lambda * \theta))) \leq ma\,x\{\alpha_{\varpi}((\theta * \lambda) * 0), \alpha_{\varpi}(0)\} = \alpha_{\varpi}(\theta * \lambda)$.

Conversely, let $\varpi = \langle\tilde{\pi}_{\varpi}, \alpha_{\varpi}\rangle$ be a cubic Q-ideal which satisfies the two conditions, then

$\tilde{\pi}_{\varpi}(\theta * \lambda) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \lambda) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\}$ and by substituting **(a)**, we obtain

$\tilde{\pi}_{\varpi}(\theta * (ℷ * (ℷ * \theta))) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\}$, and

$\alpha_{\varpi}(\theta * ℷ) \leq max\{\alpha_{\varpi}((\theta * ℷ) * \epsilon), \alpha_{\varpi}(\epsilon)\}$, substituting **(b)**, we get

$\alpha_{\varpi}(\theta * (ℷ * (ℷ * \theta))) \leq max\{\alpha_{\varpi}((\theta * ℷ) * \epsilon), \alpha_{\varpi}(\epsilon)\}$, that is

$\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-commutative ideal. □

Theorem 3.15: In Q-algebra $\aleph$, the following statements are equivalent.

(i) $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-commutative ideal of $\aleph$.

(ii) Every nonempty cubic level set of $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a commutative Q-ideal of $\aleph$.

Proof. (i) $\Longrightarrow$(ii) Assume that $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-commutative ideal of $\aleph$.

Let , $ℷ \in \aleph$, $\tilde{t} \in D[0,1]$, $r \in [0,1]$ and $U(\varpi, \tilde{t}, r) \neq \emptyset$.

Let $\theta \in U(\varpi, \tilde{t}, r)$, then $\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{\pi}_{\varpi}(\theta) \succcurlyeq \tilde{t}$ and $\alpha_{\varpi}(0) \leq \alpha_{\varpi}(\theta) \leq r$.

Thus, $0 \in U(\varpi, \tilde{t}, r)$.

Let $\theta, ℷ, \epsilon \in \aleph$ such that $((\theta * ℷ) * \epsilon) \in U(\varpi, \tilde{t}, r)$ and $\epsilon \in U(\varpi, \tilde{t}, r)$. Then,

$\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon) \succcurlyeq \tilde{t}$, $\alpha_{\varpi}(\theta * ℷ) * \epsilon) \leq r$ and $\tilde{\pi}_{\varpi}(\epsilon) \succcurlyeq \tilde{t}$, $\alpha_{\varpi}(\epsilon) \leq r$.

It follows that $\tilde{\pi}_{\varpi}(\theta * (ℷ * (ℷ * \theta))) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\} = rmin\{\tilde{t}, \tilde{t}\} = \tilde{t}$ and

$\alpha_{\varpi}(\theta * (ℷ * (ℷ * \theta))) \leq ma\,x\{\alpha_{\varpi}((\theta * ℷ) * \epsilon), \alpha_{\varpi}(\epsilon)\} = \max\{r, r\} = r$.

Therefore $((\theta * (ℷ * (ℷ * \theta))) \in U(\varpi, \tilde{t}, r)$ and by definition 2.13, then $U(\varpi, \tilde{t}, r)$ is a commutative Q-ideal.

(ii) $\Longrightarrow$(i)

Let $U(\varpi, \tilde{t}, r) \neq \varphi$ and it is a commutative Q-ideal of $\aleph$, for all $r \in [0,1]$ and $\tilde{t} \in D[0,1]$.

Assume that $\tilde{\pi}_{\varpi}(\theta) = \tilde{t}$ and $\alpha_{\varpi}(\theta) = r$, for any $\theta, ℷ \in \aleph$.

Since, $0 \in U(\varpi, \tilde{t}, r)$, then $\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{t} = \tilde{\pi}_{\varpi}(\theta)$ and $\alpha_{\varpi}(0) \leq r = \alpha_{\varpi}(\theta)$, for all $\theta \in \aleph$. Hence $(\boldsymbol{CC_1})$ hold.

For any $\theta, ℷ, \epsilon \in \aleph$, let $\tilde{t} = rmin\{\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\}$ and $r = max\{\alpha_{\varpi}((\theta * ℷ) * \epsilon), \alpha_{\varpi}(\epsilon)\}$. Then, $\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon) \succcurlyeq \tilde{t}$, $\tilde{\pi}_{\varpi}(\epsilon) \succcurlyeq \tilde{t}$ and $\alpha_{\varpi}((\theta * ℷ) * \epsilon) \leq r, \alpha_{\varpi}(\epsilon) \leq r$, that is $((\theta * ℷ) * \epsilon) \in U(\varpi, \tilde{t}, r)$ and

$\epsilon \in U(\varpi, \tilde{t}, r)$. It follows from hypothesis that $(\theta * (ℷ * (ℷ * \theta)) \in U(\varpi, \tilde{t}, r)$.

Thus $\tilde{\pi}_{\varpi}(\theta * (ℷ * (ℷ * \theta))) \succcurlyeq \tilde{t} = rmin\{\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\}$ and

$\alpha_{\varpi}(\theta * (ℷ * (ℷ * \theta))) \leq r = ma\,x\{\alpha_{\varpi}((\theta * ℷ) * \epsilon), \alpha_{\varpi}(\epsilon)\}$.

Therefore, $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-commutative ideal. □

5 Discussion and Conclusion

This study explored various types of cubic ideals in Q-algebra, specifically focusing on (the cubic Q-positive implicative, the cubic Q-implicative, and the cubic Q-commutative) ideals. It discusses several important properties of these ideals and presents a few relationships among them.

In our work, the relationship between the three ideals (cubic Q-positive implicative, cubic Q-implicative and a cubic Q-commutative ideal) is as follows:

1- In a commutative Q-algebra $\aleph$. If $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic positive Q-implicative ideal, then it is a cubic Q-implicative ideal. However, the converse does not hold.

2- There is no connection between cubic positive Q-implicative ideals and cubic Q-commutative ideals, nor is there a relationship between cubic Q-implicative ideals and cubic Q-commutative ideals.

3- Any cubic Q-positive implicative ideal of $\aleph$ is a cubic BCK-ideal, yet the converse statement is not true.

4- It has also been established that under certain conditions, a link exists between cubic BCK ideals and cubic Q-positive implicative ideals.

In the future, we may be able to study the following ideas: a cubic of soft fuzzy ideal, a cubic neutrosophic ideal and a graph of a cubic fuzzy ideal in Q-algebras, drawing on research such as [29, 30, and 31].

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Conflict of Interest

The authors declare no conflicts of interest.

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