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## Ranking of Fuzzy Numbers by using Scaling Method

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### ABSTRACT

In this paper, we presented for the first time a multidimensional scaling approach to find the scaling as well as the ranking of triangular fuzzy numbers. Each fuzzy number was represented by a row in a matrix, and then found the configuration points (scale points) which represent the fuzzy numbers in  $R^2$ . Since these points are not uniquely determined, we presented different techniques to reconfigure the points to compare them with other methods. The results showed the ability of ranking fuzzy numbers.

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Keywords: Fuzzy numbers, Ranking, Scaling method, Multidimensional scaling

### 1. Introduction

Ranking fuzzy numbers is a necessary step in making decision. Most of the presented method for ranking is not able to discriminate fuzzy numbers. Hence it is not possible to order them because they are partially ordered and cannot compare directly like real numbers <sup>[1]</sup>.

In order to rank fuzzy numbers, it is possible to convert them into real numbers and compared them by a ranking function. After the work of Zadeh (1965) <sup>[2]</sup>, many ranking methods have been introduced. First work was proposed by Jain <sup>[3]</sup>, since then many procedures are presented to do this aim. Some of ranking methods were reviewed in <sup>[4]</sup>. Chen introduced maximizing and minimizing set for ranking fuzzy numbers <sup>[5]</sup>. The idea of the mean was proposed by Dubois and Prade to rank fuzzy numbers <sup>[6]</sup>. Statistically, Lee and Li used probability measure to compare fuzzy numbers <sup>[7]</sup>.

Allahviranloo et al. applied a new distance measure and the generalized Hausdorff distance between generalized fuzzy numbers <sup>[8]</sup>. PhaniBushman and RaoPeddia used maximizing and minimizing set on the triangular fuzzy numbers which depend on the centroid point <sup>[9]</sup>. Ali et al. used a signed distance ranking to order interval-valued fuzzy numbers <sup>[10]</sup>.

In this paper, we used the classical multidimensional scaling to rank fuzzy numbers which is presented for the first time. We treated each fuzzy number as a row matrix

and used MDS technique to find the configuration points (scale points). These points represent the fuzzy number themselves in  $R^2$ . Since these points are not uniquely presented, this is the nature of MDS method. We presented different techniques to reconfigure the scale points to compare the results with other methods.

### 2. Basic Concepts and Definitions

Zadeh <sup>[2]</sup> dealt with problems that have source of vagueness. A fuzzy set is an extension of a crisp set by defining partial membership <sup>[11]</sup>.

#### Definition (1) <sup>[12]</sup>

A fuzzy set  $\tilde{A}$  of universe set  $X$  is the set of ordered pairs  $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$ , where  $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$ , where  $\mu_{\tilde{A}}(x)$  is the membership function for the fuzzy set.

#### Definition (2) <sup>[4]</sup>

A fuzzy number  $\tilde{A}$  is a fuzzy subset in which it is normal and convex with the following membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}^L(x) & \text{if } a \leq x \leq b \\ w & \text{if } b \leq x \leq c \\ \mu_{\tilde{A}}^R(x) & \text{if } c \leq x \leq d \\ 0 & \text{otherwise,} \end{cases}$$

where  $0 < w \leq 1$  is a constant,  $a, b, c, d$  are real numbers, and  $\mu_{\tilde{A}}^L: [a, b] \rightarrow [0, w]$ ,  $\mu_{\tilde{A}}^R: [c, d] \rightarrow [0, w]$  are two strictly monotonic and continuous functions from  $R$  to be closed interval  $[0, w]$ . It is customary to write fuzzy number as  $\tilde{A} = (a, b, c, d, w)$ , if  $w = 1$ , then  $\tilde{A} = (a, b, c, d, 1)$  is normalized fuzzy number, otherwise  $\tilde{A}$  is non-normal fuzzy number.

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**Definition (3)** [5]

A triangular fuzzy number (TFN)  $\tilde{A}$  is a fuzzy number with a membership function  $\mu_{\tilde{A}}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise,} \end{cases}$$

which can be denoted as a triplet  $(a_1, a_2, a_3)$ .

**Definition (4)** [13]

$\tilde{A}$  is said to be a triangular fuzzy number if its membership function is:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 < x < a_2 \\ \frac{x-a_3}{a_3-a_2}, & \text{if } a_2 < x < a_3 \\ 0, & \text{otherwise} \end{cases}$$

with  $a_1 \leq a_2 \leq a_3$ , and can be represented by  $\tilde{A} = (a_1, a_2, a_3)$ , then calculate the mean of  $\tilde{A}$  as:

$$E(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$$

**3. Ranking Fuzzy Numbers**

Ranking fuzzy numbers is an important tool in decision making, artificial intelligence, and other applications. Several studies have proposed for this aim. It is a necessary step in many applications that have fuzzy numbers. In the literatures simply called ranking methods [14]. One of the common types of the ranking method is to map fuzzy numbers into the real line. i.e., they are transformation functions  $M : F \rightarrow R$  which associate each fuzzy number with a real number and then order on the real line.

**4. Mathematics of Metric Multidimensional Scaling** [12]

This method starts with an  $(n \times n)$  matrix  $D = (\delta_{ij})$  called distance matrix. The aim is to find  $n$  points in  $k$ -dimensions such that the interpoint distances  $d_{ij}$  in the  $k$ -dimensions are approximately equal to the values of  $\delta_{ij}$  in  $D$ . Ideally  $k = 2$ . The steps are as follows

- $A = (a_{ij}) = (-\frac{1}{2}\delta_{ij}^2)$ , is an  $(n \times n)$  matrix, where  $\delta_{ij}$  is the  $ij$ th element of  $D$ .
- $B = (b_{ij})$ , is an  $(n \times n)$  matrix, where
- $b_{ij} = a_{ij} - \bar{a}_i - \bar{a}_j + \bar{a} \dots$  and
- $\bar{a}_i = \sum_{j=1}^n a_{ij}/n, \bar{a}_j = \sum_{i=1}^n a_{ij}/n$ . So

$$B = \left(I - \frac{1}{n}J\right)A\left(I - \frac{1}{n}J\right), \tag{1}$$

and

$$B = V\Lambda V', \tag{2}$$

where  $V$  is the matrix eigenvectors of  $B$  and  $\Lambda$  is the diagonal matrix of eigenvalues of  $B$ . Also it can be written as

$$B = V_1 \Lambda_1 V_1'$$

$$= V_1 \Lambda_1^{1/2} \Lambda_1^{1/2} V_1'$$

$$= ZZ',$$

where

$$Z = V_1 \Lambda_1^{1/2} (\sqrt{\lambda_1}v_1, \sqrt{\lambda_2}v_2, \dots, \sqrt{\lambda_q}v_q) \begin{pmatrix} z'_1 \\ z'_2 \\ \vdots \\ z'_n \end{pmatrix} \tag{3}$$

is our scale points.

**5. Different Techniques for Ranking**

**5.1 Scaling Method  $S_1$**

Parandin and Fariborzi [15] introduced a new idea by finding a new point differs from zero, and use a distance function. Apply it to the our points we define  $M = (\bar{z}_m, \bar{z}_p)$  such that  $\bar{z}_m$  is maximum of  $z_{i1}$  and  $\bar{z}_p$  is maximum of  $z_{i2}$ , for  $i = 1, 2, \dots, n$ . Let  $Z_{\tilde{A}_1} = (z_{11}, z_{12}), Z_{\tilde{A}_2} = (z_{21}, z_{22}), \dots, Z_{\tilde{A}_n} = (z_{n1}, z_{n2})$  be the scales of the fuzzy numbers.

Calculate the distance between  $M$  and  $Z_{\tilde{A}_i}$  as follows,

$$R(Z_{\tilde{A}_i}, M) = \sqrt{(\bar{z}_m - z_{i1})^2 + (\bar{z}_p - z_{i2})^2}$$

If  $\tilde{A}_1$  and  $\tilde{A}_2$  are two fuzzy numbers, then

1.  $R(Z_{\tilde{A}_1}, M) < R(Z_{\tilde{A}_2}, M)$  implies that  $\tilde{A}_1 > \tilde{A}_2$
2.  $R(Z_{\tilde{A}_1}, M) > R(Z_{\tilde{A}_2}, M)$  implies that  $\tilde{A}_1 < \tilde{A}_2$

**5.2 Scaling Method  $S_2$**

Allahviranloo and Saneifard [16] introduced the maximum crisp value  $\tau_{max}$  as

$$\tau_{max} = \max \{x | x \in \text{Domain}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)\}$$

and found a new point  $(\tau_{max}, 0)$ , then used a distance between centroid pinot and the new point. Applying this idea, we construct the point  $T = (\tau_{max}, 0)$ , and the scaling points are  $Z_{\tilde{A}_1} = (z_{11}, z_{12}), Z_{\tilde{A}_2} = (z_{21}, z_{22}), \dots, Z_{\tilde{A}_n} = (z_{n1}, z_{n2})$ . The ranking function is the distance between  $T$  and  $Z_{\tilde{A}_i}$  as follows:

$$R(Z_{\tilde{A}_i}, T) = \sqrt{(z_{i1} - \tau_{max})^2 + (z_{i2} - 0)^2}$$

If  $\tilde{A}_1$  and  $\tilde{A}_2$  are two fuzzy numbers, then

1.  $R(Z_{\tilde{A}_1}, T) < R(Z_{\tilde{A}_2}, T)$  implies that  $\tilde{A}_1 > \tilde{A}_2$
2.  $R(Z_{\tilde{A}_1}, T) > R(Z_{\tilde{A}_2}, T)$  implies that  $\tilde{A}_1 < \tilde{A}_2$

**5.3 A Novel Proposed Method  $S_3$**

Here, we introduce a novel method by defining the maximum crisp value  $\mu_{max}$  as follows:

$$\mu_{max} = \max\{E(\tilde{A}_1), E(\tilde{A}_2), \dots, E(\tilde{A}_n)\},$$

where  $E(\tilde{A}_i)$  is the mean value of fuzzy numbers (Definition 4). Construct the point  $E = (\mu_{max}, 0)$  and  $Z_{\tilde{A}_1} = (z_{11}, z_{12})$ ,  $Z_{\tilde{A}_2} = (z_{21}, z_{22})$ , ...,  $Z_{\tilde{A}_n} = (z_{n1}, z_{n2})$ , where  $Z_{\tilde{A}_i}$  are the scaling points for  $i = 1, 2, \dots, n$ .

Now for ranking, we calculate the distance between  $E$  and  $\tilde{A}_i$  as follows:

$$R(\tilde{A}_i, E) = \sqrt{(z_{i1} - \mu_{max})^2 + (z_{i2} - 0)^2}$$

If  $\tilde{A}_1$  and  $\tilde{A}_2$  are two fuzzy numbers, then

$$R(Z_{\tilde{A}_1}, E) < R(Z_{\tilde{A}_2}, E) \text{ implies that } \tilde{A}_1 > \tilde{A}_2$$

$$R(Z_{\tilde{A}_1}, E) > R(Z_{\tilde{A}_2}, E) \text{ implies that } \tilde{A}_1 < \tilde{A}_2$$

**Proposition (1):** If at least one of  $z_{i1} > \mu_{max}$ , then  $\exists \beta > 0$ , such that  $z_{i1} - \beta \leq \mu_{max}$  for each  $i = 1, 2, \dots, n$ .

**Proof:** Since  $z_{i1} > \mu_{max}$ , say  $i_0$  then  $z_{i_01} - \mu_{max} > 0$ . Let  $\beta = |z_{i_01} - \mu_{max}|$ . So,  $z_{i_01} - \beta \leq \mu_{max}$ . ■

For ranking fuzzy numbers, the new scale points is

$$Z^{new} = \begin{pmatrix} z_{11} - \beta & z_{12} \\ z_{21} - \beta & z_{22} \\ \vdots & \vdots \\ z_{n1} - \beta & z_{n2} \end{pmatrix}$$

### 5.4 Shifting Scale Points Techniques

It is important to mention that if the scale points are not all in  $R^+$ , so we face some shortcomes when find the distance with  $(0, 0)$  point. One of our tasks is to fix it by the following proposition.

Let

$$Z = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \\ \vdots & \vdots \\ z_{i1} & z_{i2} \end{pmatrix}$$

**Proposition (2):** If at least one of  $z_{i1} < 0$  or  $z_{i2} < 0$ , then  $\exists \epsilon_x > 0$  and  $\epsilon_y > 0$  respectively such that  $z_{i1} + \epsilon_x$  and  $z_{i2} + \epsilon_y$  are in  $R^+$ ,  $\forall i = 1, 2, \dots, n$ .

**Proof:** The goal is to find such  $\epsilon_x$  and  $\epsilon_y$ . Let  $z_{i_01} = \min(z_{i1})$ ,  $\forall i = 1, 2, \dots, n$ . For  $\epsilon_x$ ,  $z_{i_01} + \epsilon_x > 0$ , implies that  $\epsilon_x > -z_{i_01}$ , which is the lower bound of  $\epsilon_x$ .

Also, let  $z_{i_01} + \epsilon_x \leq \mu_{max}$ , implies that  $\epsilon_x \leq \mu_{max} - z_{i_01}$ , which is the upper bound of  $\epsilon_x$ , and  $\mu_{max}$  is maximum mean of the fuzzy number  $\tilde{A}_i$ . Therefore  $\epsilon_x \in (z_{i_01}, \mu_{max} - z_{i_01}]$ .

For  $\epsilon_y$ , let  $z_{i_02} = \min\{z_{i2}\}$ , so  $z_{i_02} + \epsilon_y > 0$ , implies that  $\epsilon_y > -z_{i_02}$ , which is the lower bound

of  $\epsilon_y$ . Again, let  $z_{i_02} + \epsilon_y \leq \max\{z_{i2}\}$ , implies that  $\epsilon_y \leq \max\{z_{i2}\} - z_{i_02}$ , which is the upper bound of  $\epsilon_y$ . Therefore  $\epsilon_y \in (-z_{i_02}, \max\{z_{i2}\} - z_{i_02}]$ . ■

So, in  $S_4$ , the new scale points is

$$Z^* = \begin{pmatrix} z_{11} + \epsilon_x & z_{12} + \epsilon_y \\ z_{21} + \epsilon_x & z_{22} + \epsilon_y \\ \vdots & \vdots \\ z_{n1} + \epsilon_x & z_{n2} + \epsilon_y \end{pmatrix}$$

**Remark (1):**

If all  $z_{i1} > 0$ , then no need to shift as well as for  $z_{i2}$ .

**Remark (2):**

For ranking, we choose the midpoint of  $\epsilon_x$  and  $\epsilon_y$ .

### 5.5 A Novel Proposed Method $S_4$

Define the ranking function as  $R(Z_{\tilde{A}_i}^*) = \sqrt{(z_{i1}^*)^2 + (z_{i2}^*)^2}$ , for  $\forall i$ , and  $R(Z_{\tilde{A}_i}^*)$  is the distance between point  $Z_{\tilde{A}_i}^*$  and the original point.

If  $\tilde{A}_1$  and  $\tilde{A}_2$  are two fuzzy numbers, then

$$R(Z_{\tilde{A}_1}^*) < R(Z_{\tilde{A}_2}^*) \text{ implies that } \tilde{A}_1 < \tilde{A}_2$$

$$R(Z_{\tilde{A}_1}^*) > R(Z_{\tilde{A}_2}^*) \text{ implies that } \tilde{A}_1 > \tilde{A}_2$$

**Remark (3)**

For all the proposed methods  $S_1, S_2, S_3, S_4$ . If the ranking function is equal, then it is possible to replace the position of the eigenvalues, to fix it.

### 5.6 Computational Results

In this section, we will apply our proposed methods on several instances, where some of the aforementioned methods are unable to rank them. Let

$$\text{Set 1: } \tilde{A} = (0.2, 0.3, 0.5), \tilde{B} = (0.17, 0.32, 0.58) \text{ and } \tilde{C} = (0.25, 0.4, 0.7)$$

$$\text{Set 2: } \tilde{A} = (2, 4, 6), \tilde{B} = (1, 5, 6) \text{ and } \tilde{C} = (3, 5, 6)$$

$$\text{Set 3: } \tilde{A} = (0.1, 0.2, 0.3), \tilde{B} = (0.15, 0.26, 0.32) \text{ and } \tilde{C} = (0.15, 0.3, 0.4)$$

$$\text{Set 4: } \tilde{A} = (1, 2, 5), \tilde{B} = (0, 3, 4) \text{ and } \tilde{C} = (2, 2.5, 3)$$

$$\text{Set 5: } \tilde{A} = (0, 1, 3), \tilde{B} = (4, 5, 6) \text{ and } \tilde{C} = (7, 8, 9)$$

Set 6:  $\tilde{A} = (1, 1, 5)$ ,  $\tilde{B} = (0.25, 1, 2)$  and  $\tilde{C} = (1, 2, 9)$ .

Table 1 shows the results of the proposed methods

**Table 1:** Results for the (set 1, set 2, set 3)

Authors	Fuzzy Number	Set 1	Set 2	Set 3
Cheng distance <sup>[17]</sup>	$\tilde{A}$	0.5900	4.0311	0.5385
	$\tilde{B}$	0.6040	4.0348	0.5636
	$\tilde{C}$	0.6620	4.6943	0.5810
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Parandin and Fariborzi <sup>[15]</sup>	$\tilde{A}$	0.3420	0.6672	0.0838
	$\tilde{B}$	0.1170	0.6666	0.0400
	$\tilde{C}$	0.0015	0.0206	0.0012
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Allahviranloo and Saneifard <sup>[16]</sup>	$\tilde{A}$	0.6097	2.0615	0.5385
	$\tilde{B}$	0.5955	2.0688	0.5320
	$\tilde{C}$	0.5462	1.4271	0.5205
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Chu and Tsao <sup>[18]</sup>	$\tilde{A}$	0.1620	2.0000	0.1000
	$\tilde{B}$	0.1740	2.1176	0.1237
	$\tilde{C}$	0.2190	2.3742	0.1437
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Asady and Zendehnam <sup>[19]</sup>	$\tilde{A}$	0.2500	2.2500	0.1250
	$\tilde{B}$	0.2275	1.2500	0.1650
	$\tilde{C}$	0.3250	3.2500	0.1750
Results	$\tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Phani and Ravi <sup>[11]</sup>	$\tilde{A}$	0.5180	4.1036	0.4591
	$\tilde{B}$	0.5266	4.5924	0.4847
	$\tilde{C}$	0.5847	4.3402	0.5080
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Yager <sup>[20]</sup>	$\tilde{A}$	0.3250	4.0000	0.2000
	$\tilde{B}$	0.3475	4.2500	0.2475
	$\tilde{C}$	0.4375	4.7500	0.2875
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Wang and Luo <sup>[21]</sup>	$\tilde{A}$	0.2925	0.6000	0.3333
	$\tilde{B}$	0.3349	0.6500	0.4917
	$\tilde{C}$	0.5047	0.7500	0.6294
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Chen <sup>[5]</sup>	$\tilde{A}$	0.3706	0.5516	0.4098
	$\tilde{B}$	0.3997	0.6140	0.5072
	$\tilde{C}$	0.4828	0.6525	0.5753
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Chang <sup>[22]</sup>	$\tilde{A}$	0.0166	4.0000	0.0100
	$\tilde{B}$	0.0267	8.0000	0.0133
	$\tilde{C}$	0.3375	4.3333	0.0212
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$
$S_1$ method	$\tilde{A}$	0.2350	1.0000	0.1500
	$\tilde{B}$	0.1597	2.2368	0.0902

Results	$\tilde{C}$	0.0414 $\tilde{A} < \tilde{B} < \tilde{C}$	0.3333 $\tilde{B} < \tilde{A} < \tilde{C}$	0.0020 $\tilde{A} < \tilde{B} < \tilde{C}$
$S_2$ method	$\tilde{A}$ $\tilde{B}$	0.7996 0.7308 0.5705	6.0369 7.0079 5.0110	0.4734 0.4045 0.3234
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
$S_3$ method	$\tilde{A}$ $\tilde{B}$	0.5247 0.4563 0.2956	4.8790 5.8428 3.8477	0.3650 0.2964 0.2151
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
$S_4$ method	$\tilde{A}$ $\tilde{B}$	0.2130 0.2901 0.4422	4.3687 2.4394 4.4291	0.1534 0.2118 0.2993
Results	$\tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$

Table 2: Results for the (set 4, set 5, set 6)

Authors	Fuzzy Number	Set 4	Set 5	Set 6
Cheng distance <sup>[17]</sup>	$\tilde{A}$ $\tilde{B}$	2.7071 2.3935 2.5495	1.4126 5.0249 8.0156	2.3655 1.1890 4.0310
Results	$\tilde{C}$	$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Parandin and Fariborzi <sup>[15]</sup>	$\tilde{A}$ $\tilde{B}$	0.0666 0.3333 0.1700	6.6667 3.0000 0.0000	1.6704 2.9265 0.0000
Results	$\tilde{C}$	$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Allahviranloo and Saneifard <sup>[16]</sup>	$\tilde{A}$ $\tilde{B}$	2.3500 2.6800 2.5200	7.6808 4.0311 1.1180	6.6779 7.9318 5.0249
Results	$\tilde{C}$	$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Chu and Tsao <sup>[18]</sup>	$\tilde{A}$ $\tilde{B}$	1.2444 1.2444 1.2500	0.6222 2.5000 4.0000	0.9074 0.5310 2.0000
Results	$\tilde{C}$	$\tilde{A} \sim \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Asady and Zendehnam <sup>[19]</sup>	$\tilde{A}$ $\tilde{B}$	1.7500 0.2500 2.0625	0.5000 4.2500 7.2500	0.5000 2.0000 2.7500
Results	$\tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Phani and Ravi <sup>[11]</sup>	$\tilde{A}$ $\tilde{B}$	2.4051 2.7296 2.5279	1.1931 5.0006 8.0004	1.0576 1.7178 3.5599
Results	$\tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
Yager <sup>[20]</sup>	$\tilde{A}$ $\tilde{B}$	2.5000 2.5000 2.5000	1.2500 5.0000 8.0000	2.0000 1.0625 3.500
Results	$\tilde{C}$	$\tilde{A} \sim \tilde{B} \sim \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$

Wang and Luo <sup>[21]</sup>	$\tilde{A}$	0.5000	0.1389	0.2000
	$\tilde{B}$	0.5000	0.5558	0.0929
	$\tilde{C}$	0.4667	0.8889	0.3714
	Results	$\tilde{C} < \tilde{A} \sim \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Chen <sup>[5]</sup>	$\tilde{A}$	0.4720	0.2654	0.2933
	$\tilde{B}$	0.5280	0.5355	0.2178
	$\tilde{C}$	0.5000	0.7634	0.3881
	Results	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
Chang <sup>[22]</sup>	$\tilde{A}$	1.3333	0.6666	0.0000
	$\tilde{B}$	3.5000	2.5000	0.4062
	$\tilde{C}$	0.3125	4.0000	2.0000
	Results	$\tilde{C} < A < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$
S <sub>1</sub> method	$\tilde{A}$	2.1213	11.5758	4.0833
	$\tilde{B}$	2.7385	5.1961	7.1384
	$\tilde{C}$	0.8660	0.0000	0.0578
	Results	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
S <sub>2</sub> method	$\tilde{A}$	4.1940	14.9900	9.3605
	$\tilde{B}$	5.9084	8.5997	12.3813
	$\tilde{C}$	5.1961	3.4165	5.2714
	Results	$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
S <sub>3</sub> method	$\tilde{A}$	1.9344	13.9901	4.1029
	$\tilde{B}$	3.6026	7.6002	7.1125
	$\tilde{C}$	3.0183	2.4176	0.1707
	Results	$\tilde{B} < \tilde{C} < \tilde{A}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$
S <sub>4</sub> method	$\tilde{A}$	2.5426	4.0226	4.5286
	$\tilde{B}$	1.4018	10.3936	1.6754
	$\tilde{C}$	3.6925	15.5823	8.6378
	Results	$\tilde{B} < \tilde{A} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{B} < \tilde{A} < \tilde{C}$

In Table 1 and set 1, the proposed methods give  $\tilde{A} < \tilde{B} < \tilde{C}$ , as in most of the methods. For set 2, they give  $\tilde{B} < \tilde{A} < \tilde{C}$ , where, most of other methods give  $\tilde{A} < \tilde{B} < \tilde{C}$ , and for set 3, they give  $\tilde{A} < \tilde{B} < \tilde{C}$  as in all the methods.

In Table 2, for set 4, the proposed methods are capable to recognize these numbers, S<sub>1</sub>, S<sub>4</sub> give  $\tilde{B} < \tilde{A} < \tilde{C}$ , and S<sub>2</sub>, S<sub>3</sub> give  $\tilde{B} < \tilde{C} < \tilde{A}$ , whereas some of other methods couldn't rank the numbers such as Yager <sup>[20]</sup>, and Chu and Tsao <sup>[18]</sup>. The other methods give different results.

For set 5, all the proposed methods and others give  $\tilde{A} < \tilde{B} < \tilde{C}$ ,

and for set 6, the proposed methods give and others give  $\tilde{B} < \tilde{A} < \tilde{C}$  as most of the other methods.

## 6. Conclusion

In this paper we proposed several approaches to find a scale of triangular fuzzy numbers, and ranking them by scaling method. The idea is presented for the first time, and has a simple structure. The scale points can be reconfigured by many techniques to fit the original distances. An important property is that, this method ranks all the triangular fuzzy numbers, unless they are equal.

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